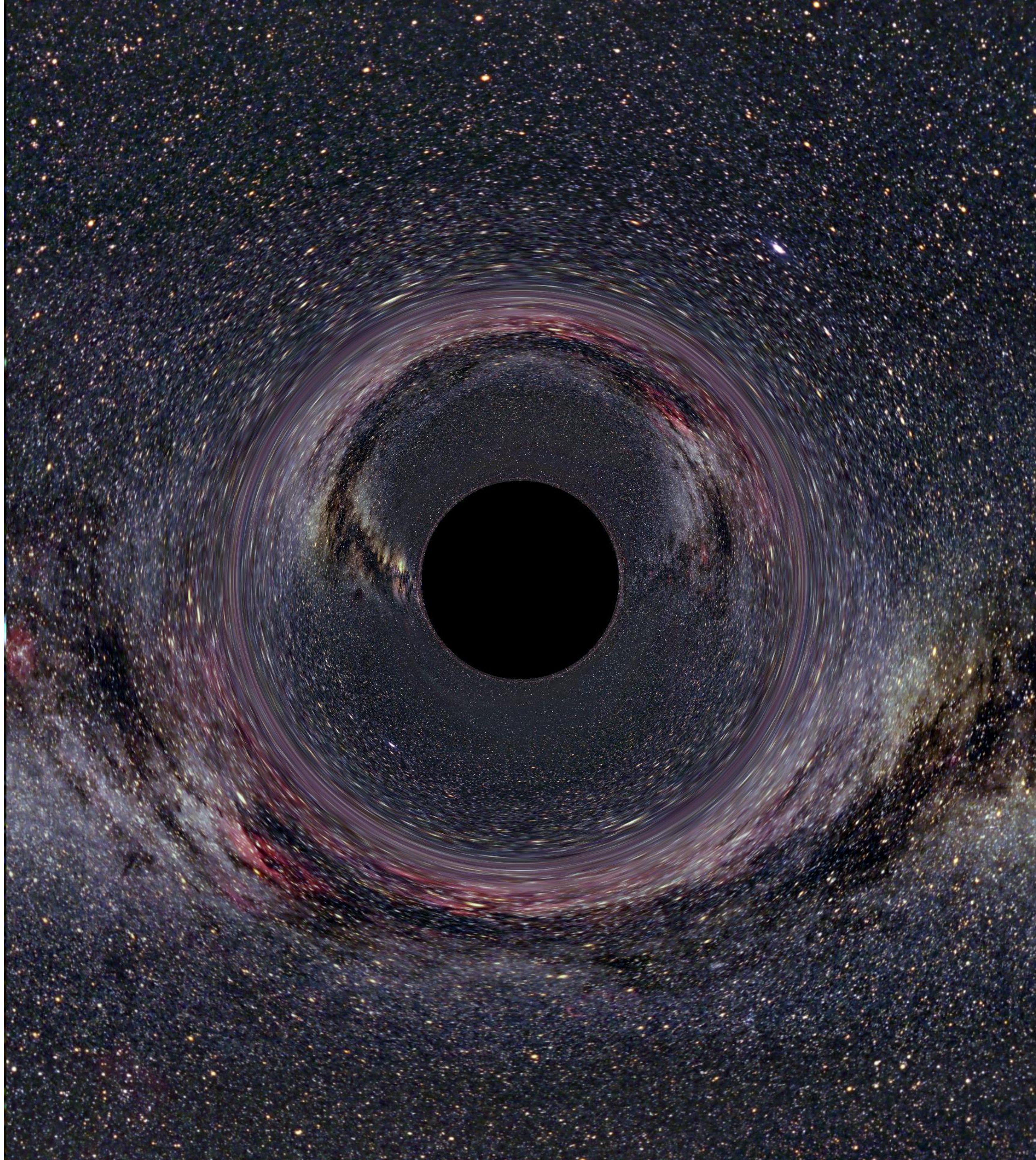
PHY 4905 | Modern Astrophysics

Lecture IX.

Black holes

Imre Bartos | Fall 2019



Homework

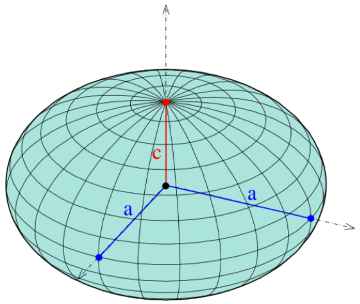
$$E_{\mathrm{GW}} \approx 10^{-2} M_{\odot} c^2 \left(\frac{\epsilon}{0.2}\right)^2 \left(\frac{f}{2\,\mathrm{kHz}}\right)^6 \left(\frac{M}{1.4\,M_{\odot}}\right) \left(\frac{R}{12\,\mathrm{km}}\right)^2 \left(\frac{\tau}{0.1\,\mathrm{s}}\right)$$

Assume realistic dependence of epsilon on f.

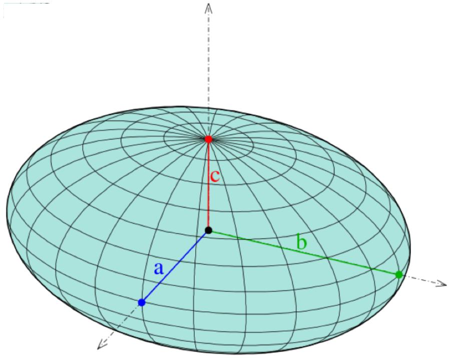
What will be the gravitational waveform?

Rotating Spheroids

MacLaurin spheroids

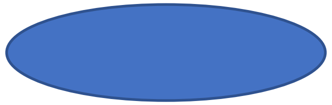


Jacobi spheroids
(or Dedekind spheroids)



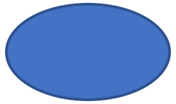
Dynamically unstable:

- T/|W| > 2.7
- e > 0.95 (1:3:3)



Secular instability:

- T/|W| > 1.4
- e > 0.81 (1:1.7)



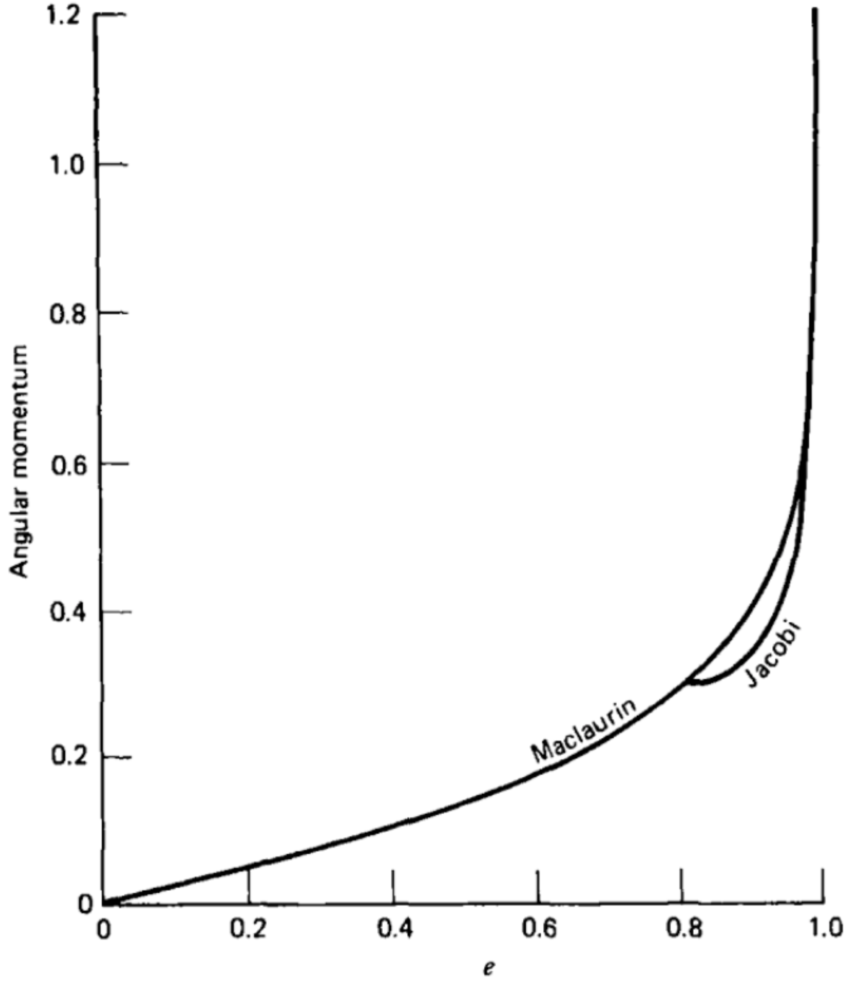
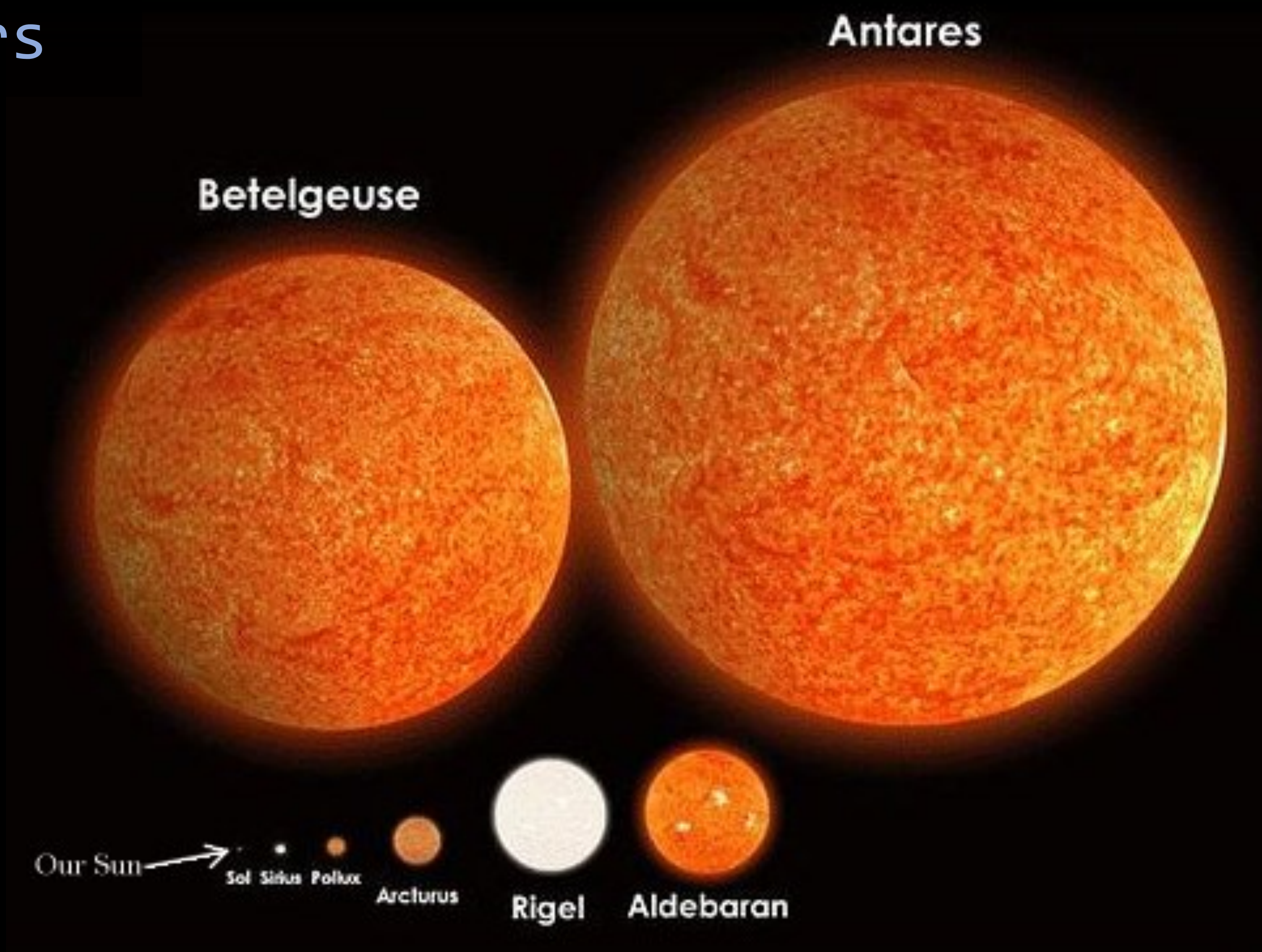


Figure 7.3 The angular momentum [in the unit $(GM^3\bar{a})^{1/2}$] along the Maclaurin and the Jacobian sequences. Here $\bar{a}$ is related to the three semimajor axes by $\bar{a} \equiv (abc)^{1/3}$; for a Maclaurin spheroid $a = b$. The abscissa in both cases is the eccentricity defined by Eq. (7.3.9). [Reproduced, with permission, from *Ellipsoidal Figures of Equilibrium* by S. Chandrasekhar, published by Yale University Press. © 1969 by Yale University.]

dark stars



John Michell (1783)

Mitchell: Escape velocity for sufficiently massive+small stars would escape the speed of light.

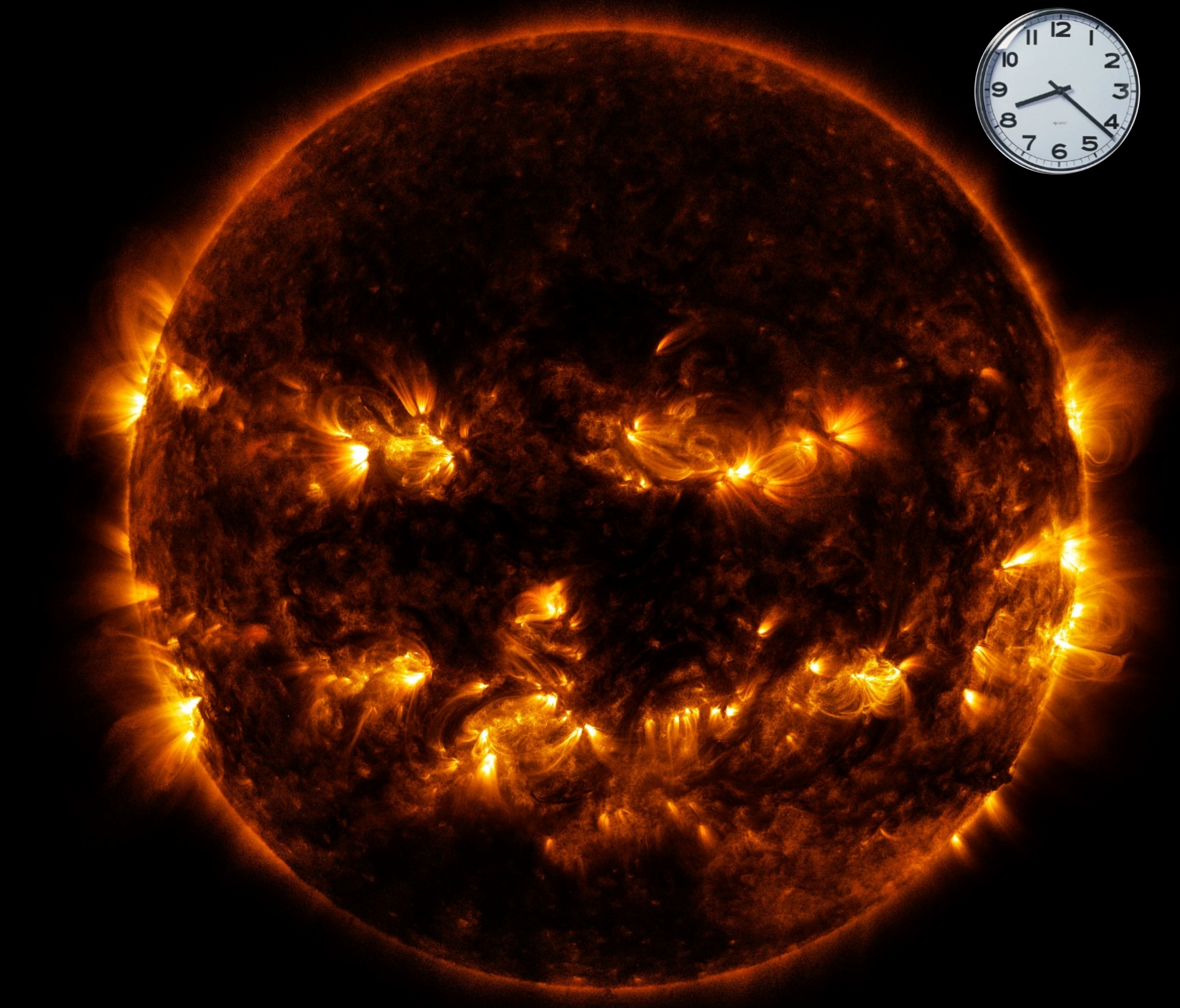
Eddington: showed that very large stars like Betelgeuse cannot possibly have the density of the Sun.

Escape velocity

At what radius would an object with the mass of the Sun become “dark?”
(assuming Newtonian gravity)

$$v_e = \sqrt{\frac{2GM}{r}}$$

“frozen stars”



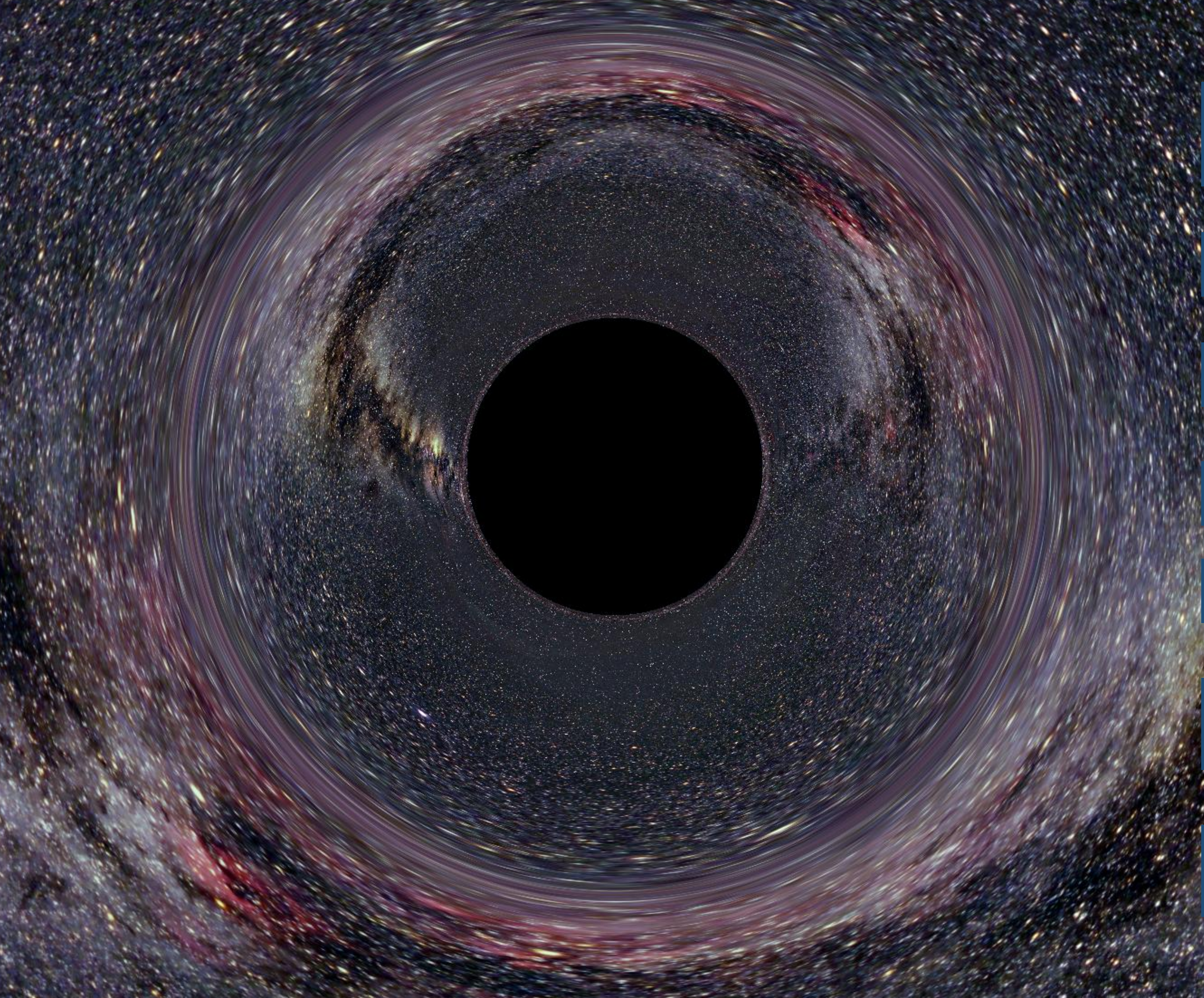
There is a mass limit above which gravitational collapse is inevitable. (Chandrasekhar 1931 for white dwarfs)

I think that there should be a law of Nature to prevent the star from behaving in this absurd way. (Eddington 1935)

…all stars heavier than 1SM, certainly possess regions in which the laws of quantum mechanics (and therefore quantum statistics) are violated.

There is a mass limit above which gravitational collapse is inevitable. (Tolman–Oppenheimer-Volkoff in 1939 for neutron stars)

→ Time would stop for the collapse at the Schwarzschild radius → frozen stars



1962 – discovery of compact X-ray sources.

1963 – discovery of quasars.

1963 - Kerr solution.

1965 - Newman solution.

1968 – discovery of pulsars.

1968 - Wheeler coins the name “black hole.”

1970’s – observation of X-ray binary Cygnus X-1.

No-hair theorem

A stable BH only has 3 independent physical properties:

1. mass
2. angular momentum
3. charge.

These properties are observable from outside.

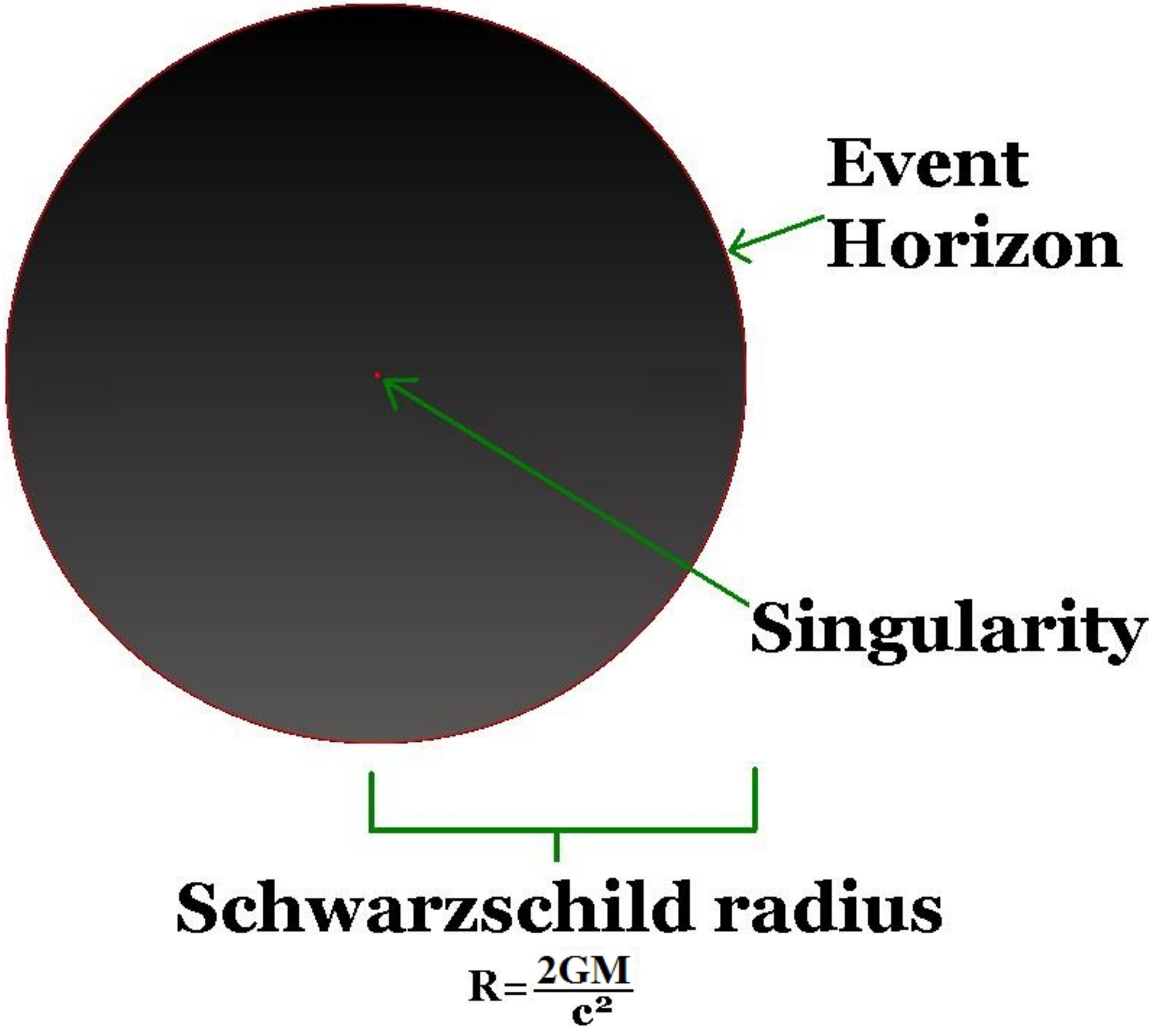
→ Loss of information upon infall.

Only mass → Schwarzschild BH
Mass + angular momentum → Kerr BH
Mass + charge → Reissner-Norström BH
Mass + angular mom. + charge → Kerr-Newman BH

Far from the horizon, the BH grav. field is identical to that of any other object with the same mass.

Limit on angular momentum and charge:

$$Q^2 + \left(\frac{J}{M}\right)^2 \leq M^2$$



(Wikipedia)

Innermost stable circular orbit (ISCO)

Schwarzschild BH:

$$r_{isco} = 3\,r_s = \frac{6\,GM}{c^2}$$

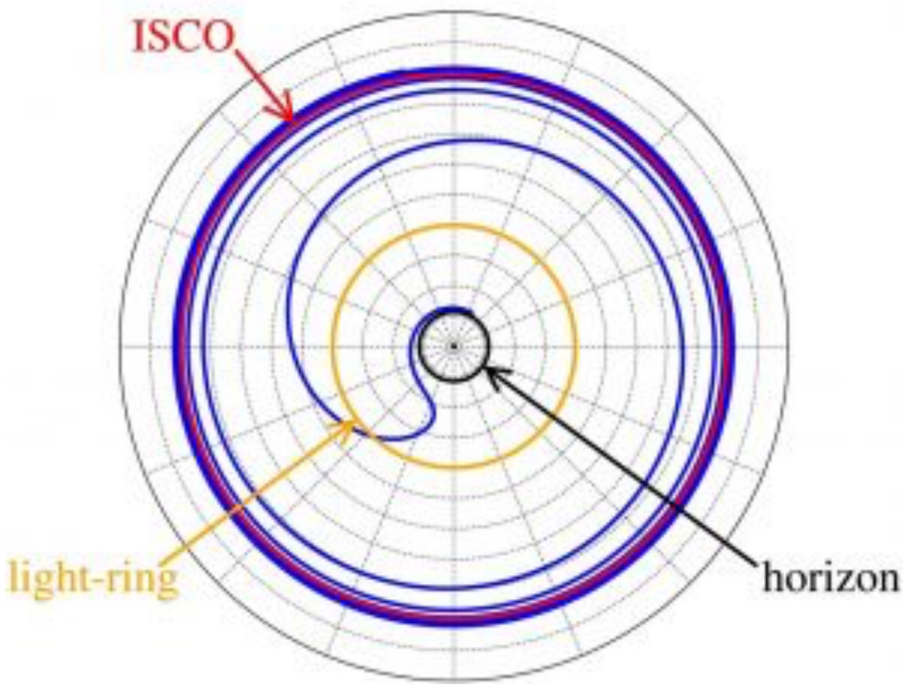
For photons (photon sphere):

$$r = \frac{3\,GM}{c^2}$$

Extreme Kerr BH (maximum spin):

$$r_{\text{ISCO}} = 9M$$ *(counter-rotating)*

$$r_{\text{ISCO}} = M$$ *(co-rotating)*



Surface area

No physical process can decrease the surface area of a BH (or multiple BHs)

Rotating BH (no charge):

$$A = 8\pi M(M + \sqrt{M^2 - J^2/M^2})$$

(→ 29% of initial mass can in principle be radiated away)

Reducing the angular momentum <u>increases</u> the surface area.

Merger of two (non-rotating BHs):

$$A \geq A_1 + A_2, \text{ or } M_f^2 \geq M_1^2 + M_2^2$$

(→ 29% of initial mass can in principle be radiated away)

Merger of two spinning black holes: up to almost <u>half</u> of mass can be extracted!

http://www.physics.umd.edu/grt/taj/776b/lectures.pdf

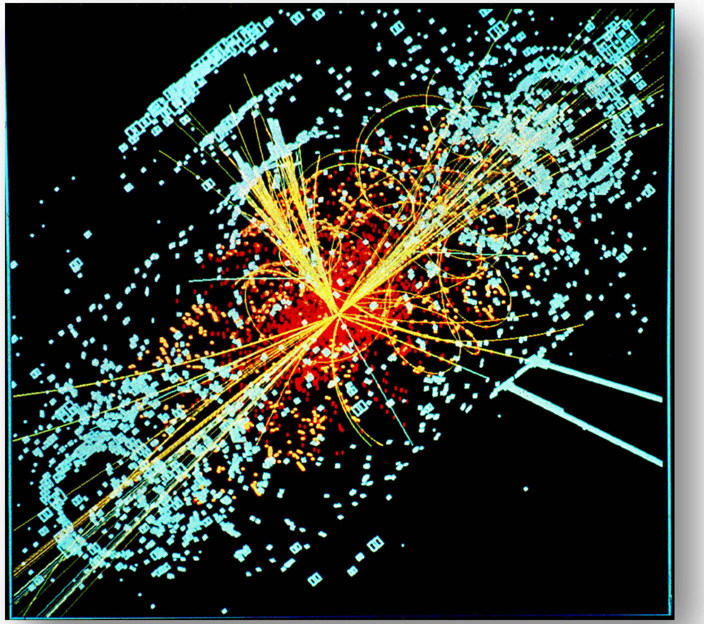
Formation mechanisms?

Micro-black holes

Schwarzschild radius: $R = 2GM/c^2$

Compton wavelength: $\lambda = h/Mc$

$$\left.\begin{array}{l} R = 2GM/c^2 \\ \lambda = h/Mc \end{array}\right\} M \geq Planck\ mass$$



In some extensions of present physics (e.g. string theory) gravity can increase faster at short distances → lower minimum BH mass

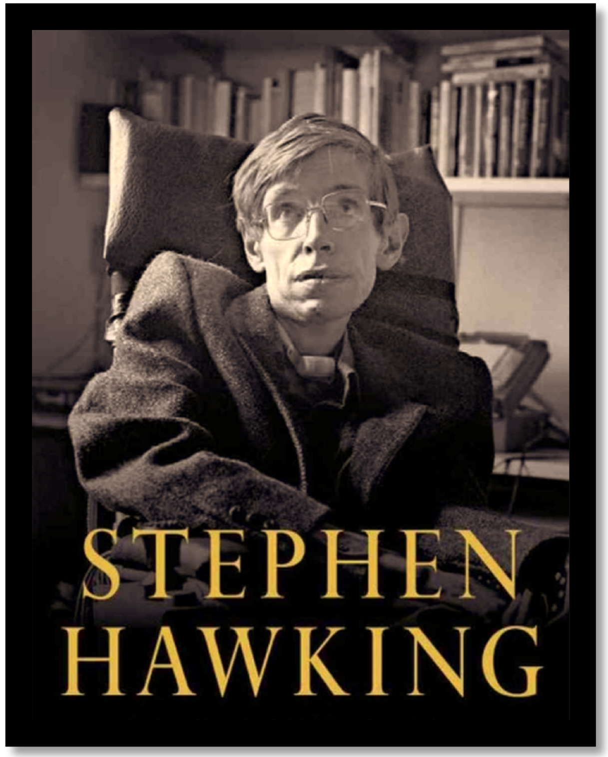
The Large Hadron Collider (LHC) and cosmic rays could produce BHs!

If such a small BH was created on Earth, would it be dangerous?

Hawking radiation

Black hole "temperature":

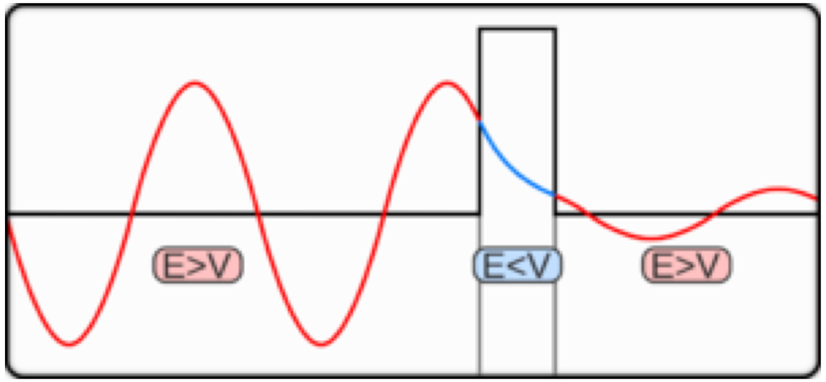
$$T = \frac{\hbar c^3}{8\pi G M k_{\mathrm{B}}} \qquad \left(\approx \frac{1.227\times 10^{23}\ \mathrm{kg}}{M}\ \mathrm{K} = 6.169\times 10^{-8}\ \mathrm{K}\times\frac{M_\odot}{M}\right)$$



Black hole will emit black body radiation at this temperature.

Irrelevant for astrophysical BHs.

Relevant for BH masses below 10^{12} kg.

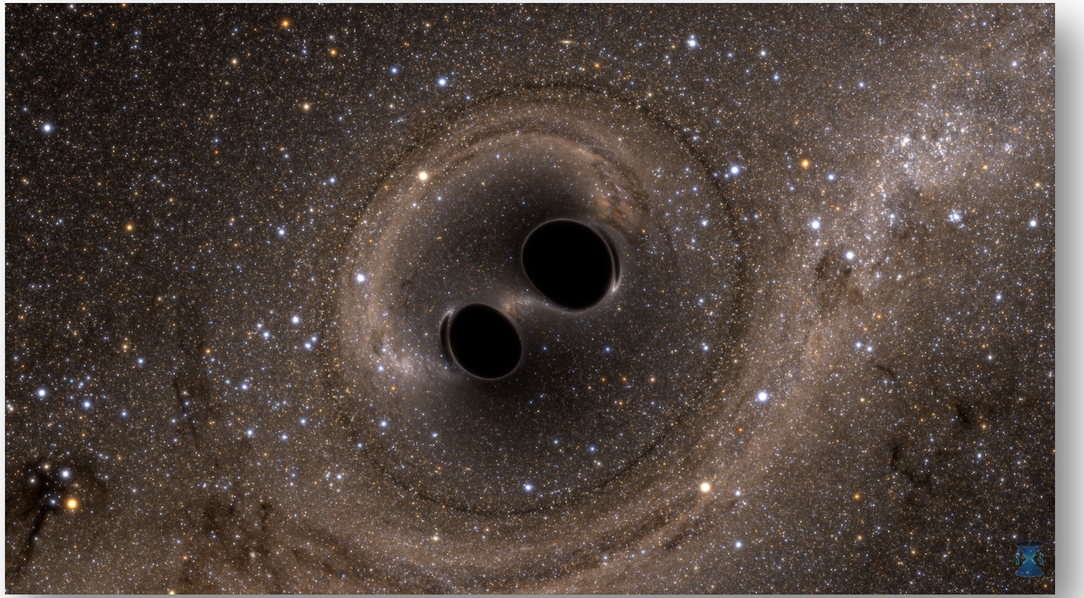


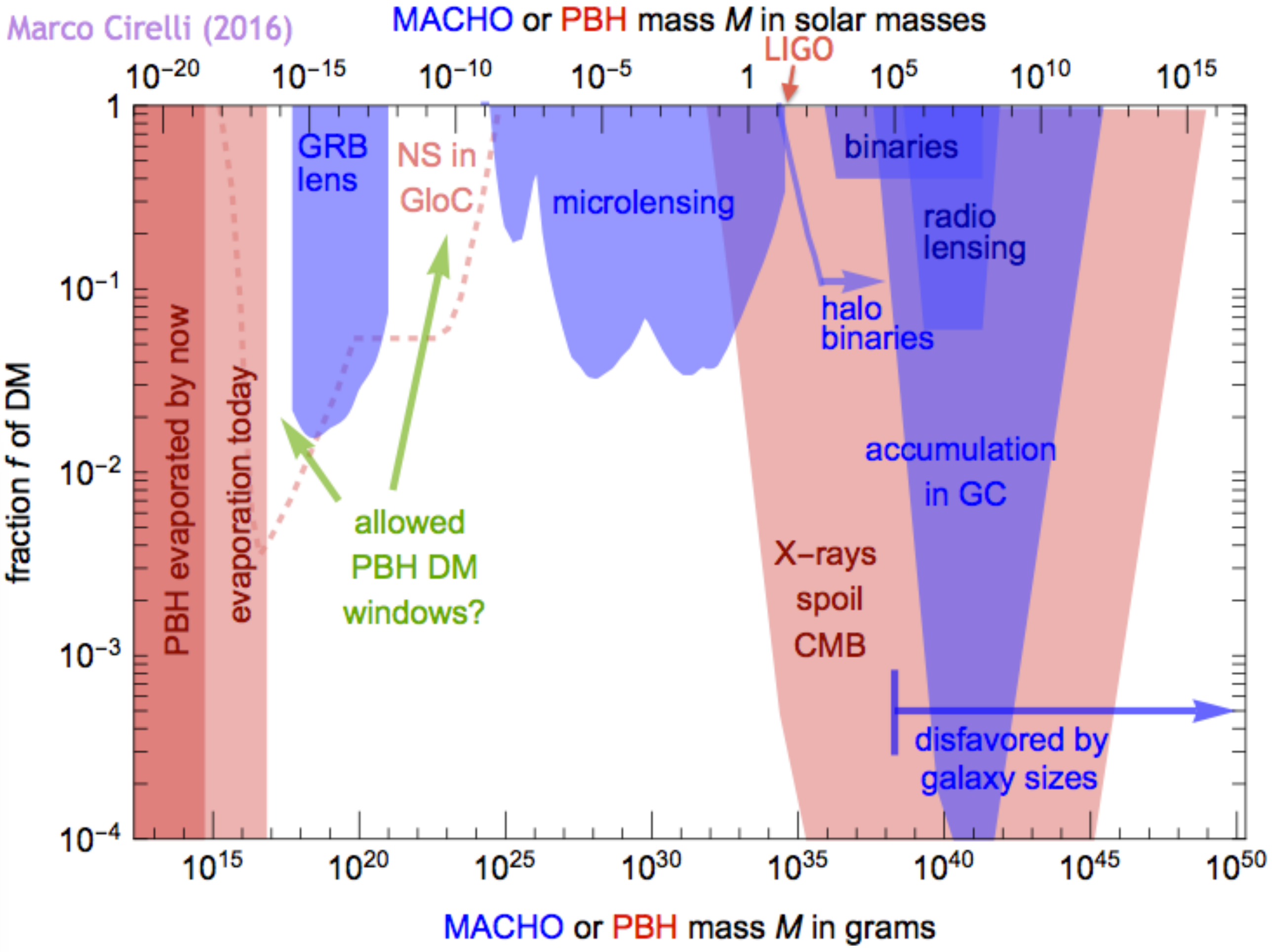
Primordial black holes

Large densities and density fluctuations soon after the big bang.

Mass distribution should be different from that of astrophysical black holes.

Primordial BHs were suggested as Dark Matter, and the origin of some LIGO BBH mergers, e.g. GW150914 (Bird+ 2016)





Black hole mass distribution

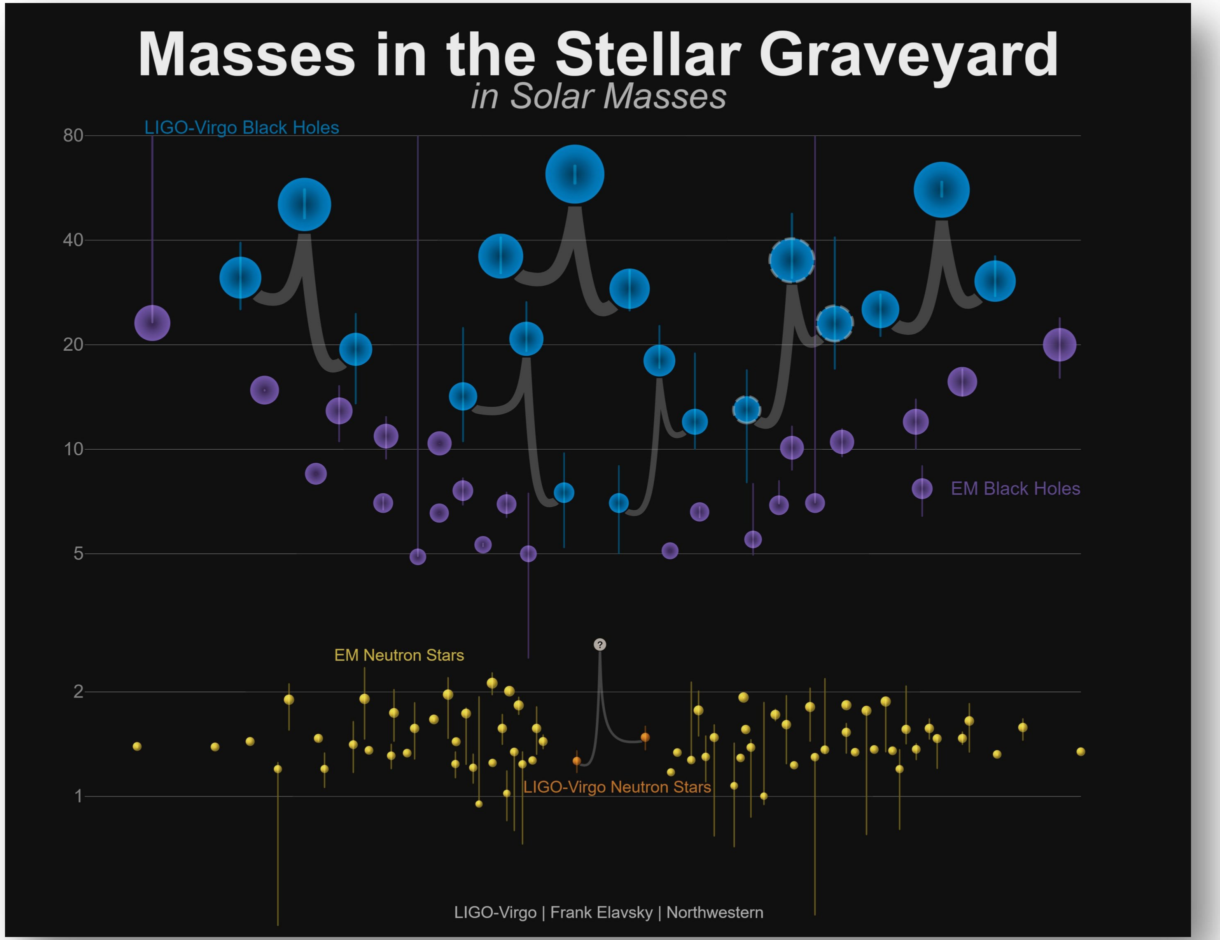
Three classes with different origin / evolution

- Stellar-mass (3 Msun – ~100 Msun)
- Intermediate-mass (~100 Msun – 10^5 Msun)
- Supermassive (10^5 Msun – 10^{11} Msun)

Stellar mass mass distribution:

- We don't know
- There seems to be a mass lower limit at 5 Msun
  → mass gap
- Best guess – Salpeter function (PDF ~ $M^{-2.3}$)

LIGO's detected BH mass distribution is consistent with Salpeter function up to a cutoff mass of ~ 50 Msun



Intermediate mass black holes

No confirmed observation.

Claims:
- Measurement of Doppler shift of stellar radiation in X-ray binaries.
- Super-Eddington radiation in X-ray binaries.
- Stellar dynamics in globular clusters.
- …

LIGO has limits on their abundance.

Origin:
- From stellar mass BHs through accretion
- Collision of multiple stars or stellar remnants in dense environments
- Primordial BHs
- Collapse of Pop III stars

Supermassive black holes