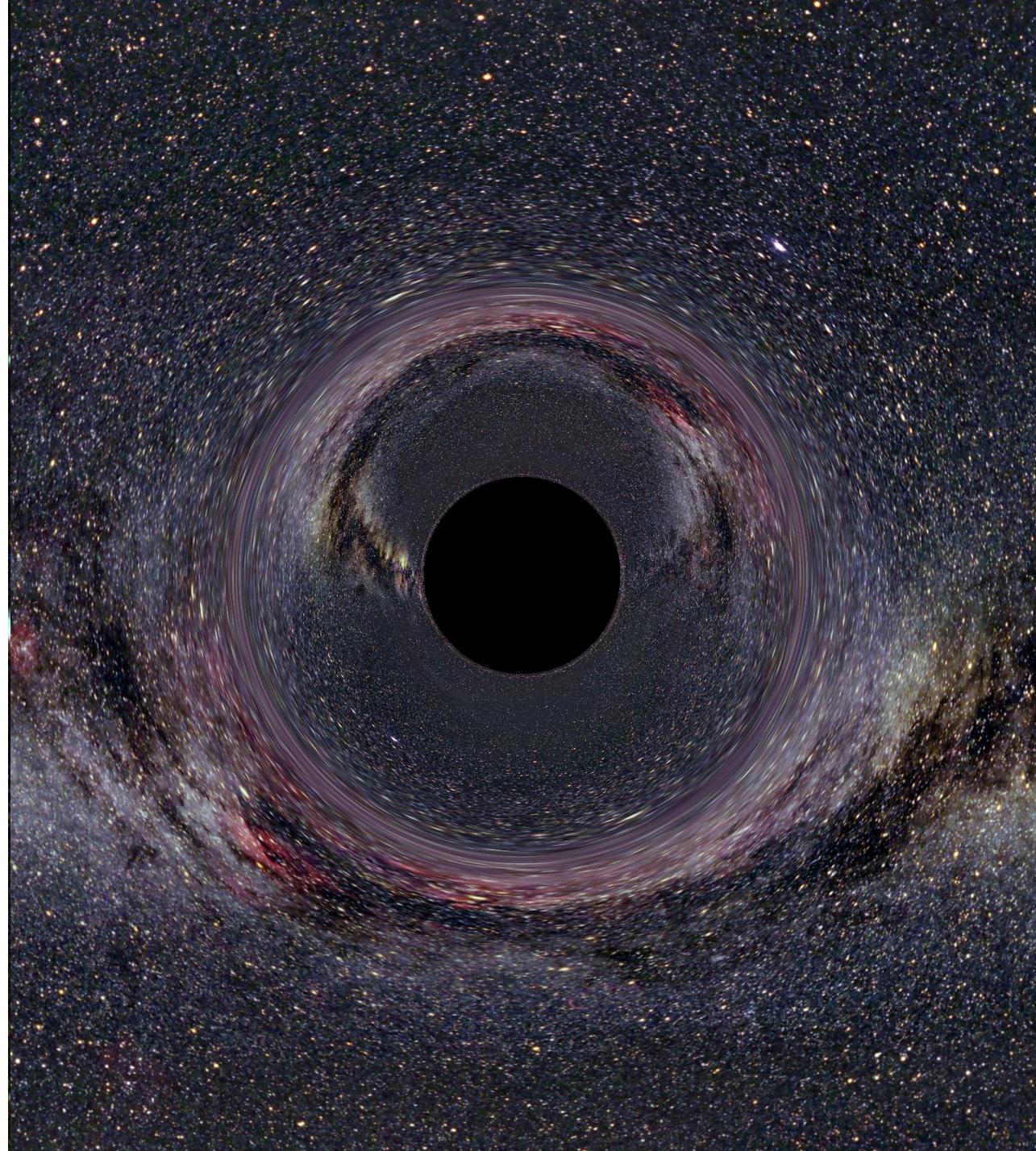


Lecture XII.

# Black holes



Imre Bartos | Fall 2018



# No-hair theorem

A stable BH only has 3 independent physical properties:

1. mass
2. angular momentum
3. charge.

These properties are observable from outside.

→ Loss of information upon infall.

Only mass → Schwarzschild BH

Mass + angular momentum → Kerr BH

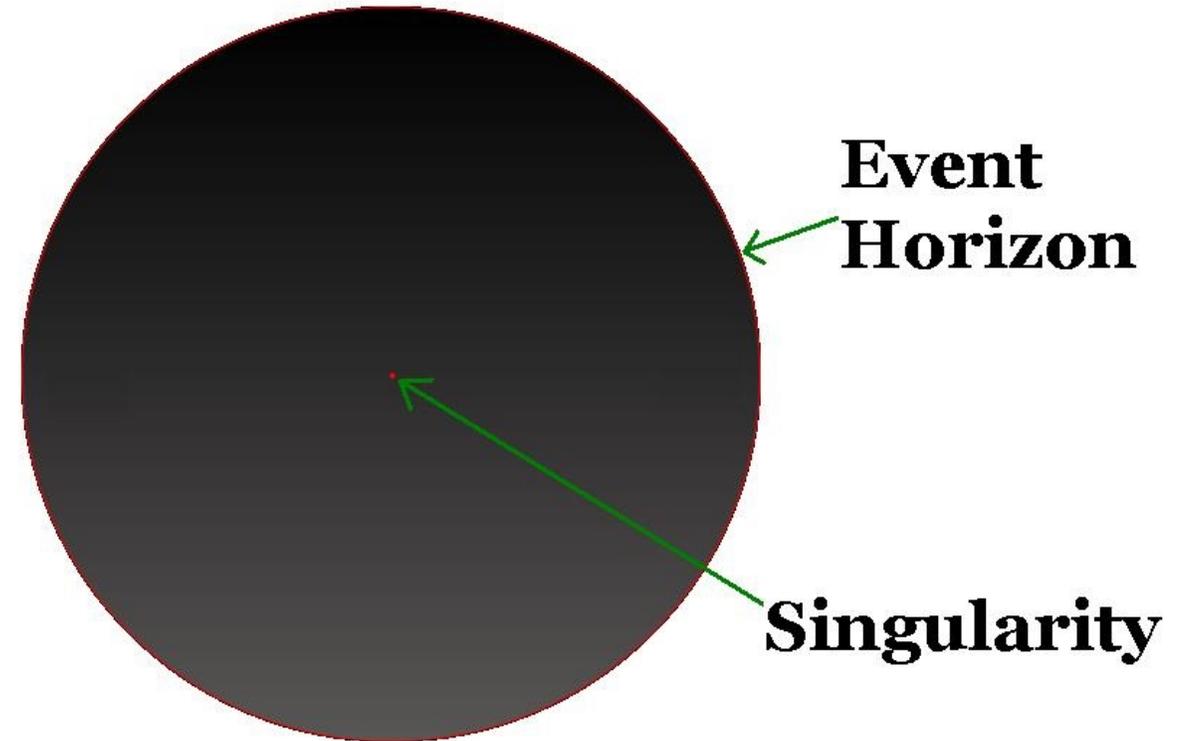
Mass + charge → Reissner-Norström BH

Mass + angular mom. + charge → Kerr-Newman BH

Far from the horizon, the BH grav. field is identical to that of any other object with the same mass.

Limit on angular momentum and charge:

$$Q^2 + \left(\frac{J}{M}\right)^2 \leq M^2$$



**Schwarzschild radius**

$$R = \frac{2GM}{c^2}$$

(Wikipedia)

# Innermost stable circular orbit (ISCO)

Schwarzschild BH:

$$r_{isco} = 3 r_s = \frac{6 GM}{c^2}$$

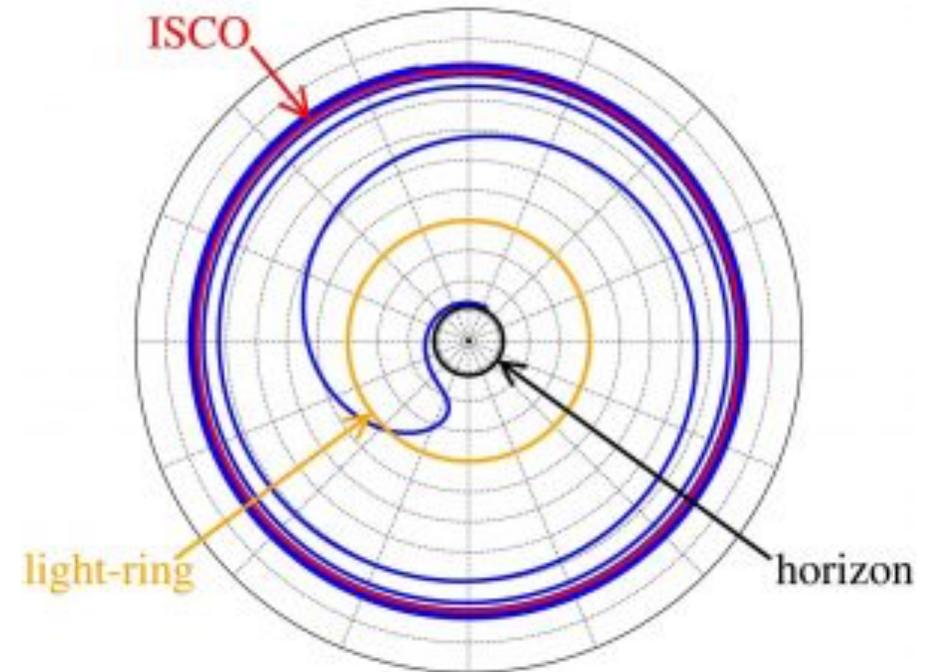
For photons (photon sphere):

$$r = \frac{3 GM}{c^2}$$

Extreme Kerr BH (maximum spin):

$$r_{ISCO} = 9M \quad (\text{counter-rotating})$$

$$r_{ISCO} = M \quad (\text{co-rotating})$$



# Surface area

No physical process can decrease the surface area of a BH (or multiple BHs)

Rotating BH (no charge):

$$A = 8\pi M(M + \sqrt{M^2 - J^2/M^2}) \quad (\rightarrow 29\% \text{ of initial mass can in principle be radiated away})$$

Reducing the angular momentum increases the surface area.

Merger of two (non-rotating BHs):

$$A \geq A_1 + A_2, \text{ or } M_f^2 \geq M_1^2 + M_2^2$$

( $\rightarrow$  29% of initial mass can in principle be radiated away)

Merger of two spinning black holes: up to almost half of mass can be extracted!

# Formation mechanisms?

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# Micro-black holes

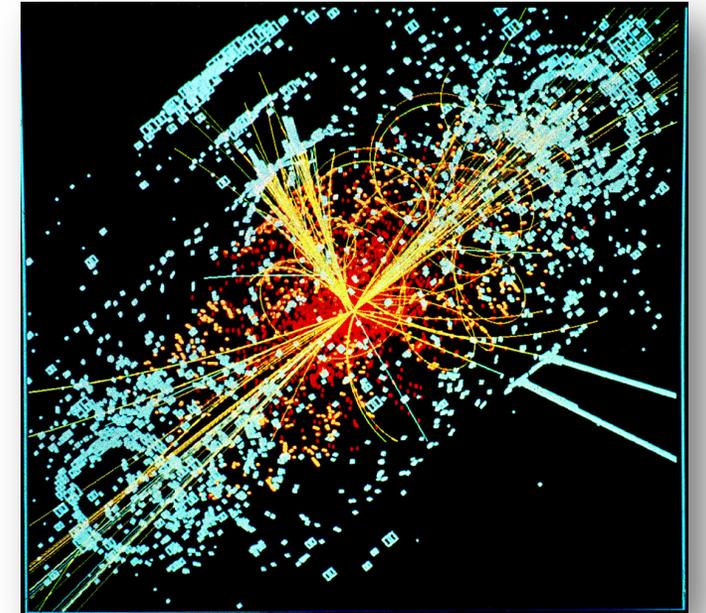
Schwarzschild radius:  $R = 2GM/c^2$

Compton wavelength:  $\lambda = h/Mc$

$M \geq \text{Planck mass}$   
( $10^{-8}$  kg)

In some extensions of present physics (e.g. string theory) gravity can increase faster at short distances  $\rightarrow$  lower minimum BH mass

 The Large Hadron Collider (LHC) and cosmic rays could produce BHs!



If such a small BH was created on Earth, would it be dangerous?

# Hawking radiation

Black hole “temperature”:

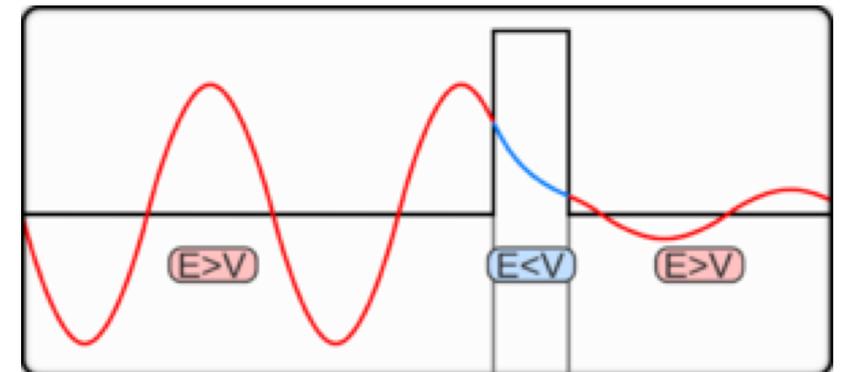
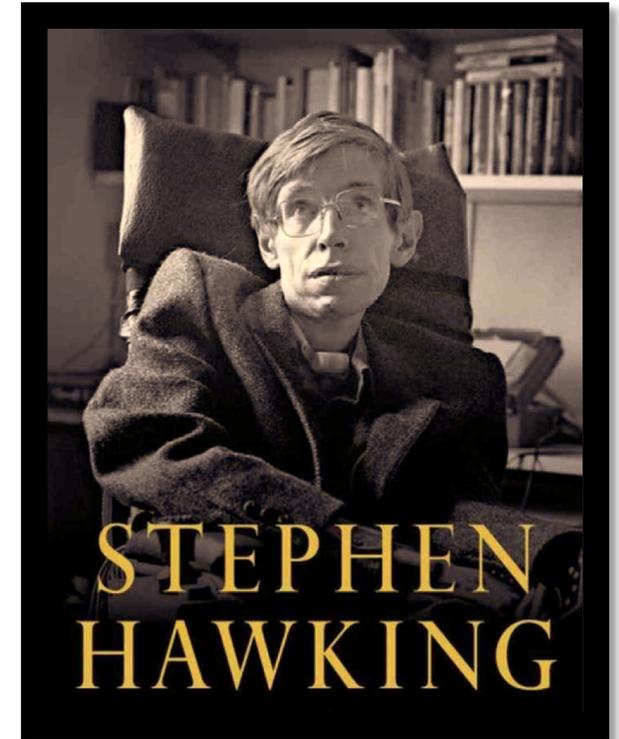
$$T = \frac{\hbar c^3}{8\pi G M k_B} \left( \approx \frac{1.227 \times 10^{23} \text{ kg}}{M} \text{ K} = 6.169 \times 10^{-8} \text{ K} \times \frac{M_\odot}{M} \right)$$

Black hole will emit black body radiation at this temperature.

Irrelevant for astrophysical BHs.

Relevant for BH masses below  $10^{12}$  kg.

HW: What is the heaviest BH that, created on Earth, you would survive?

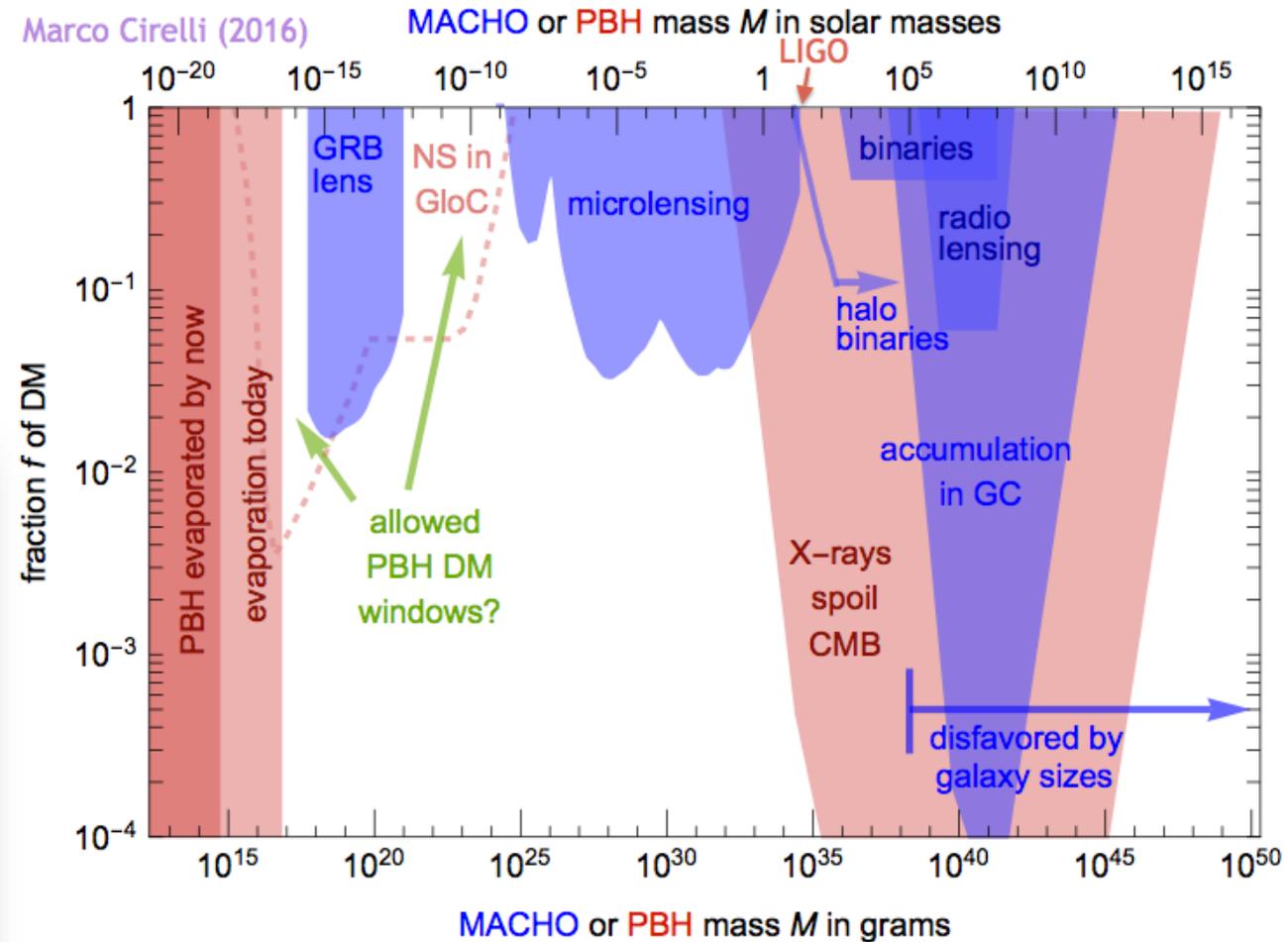
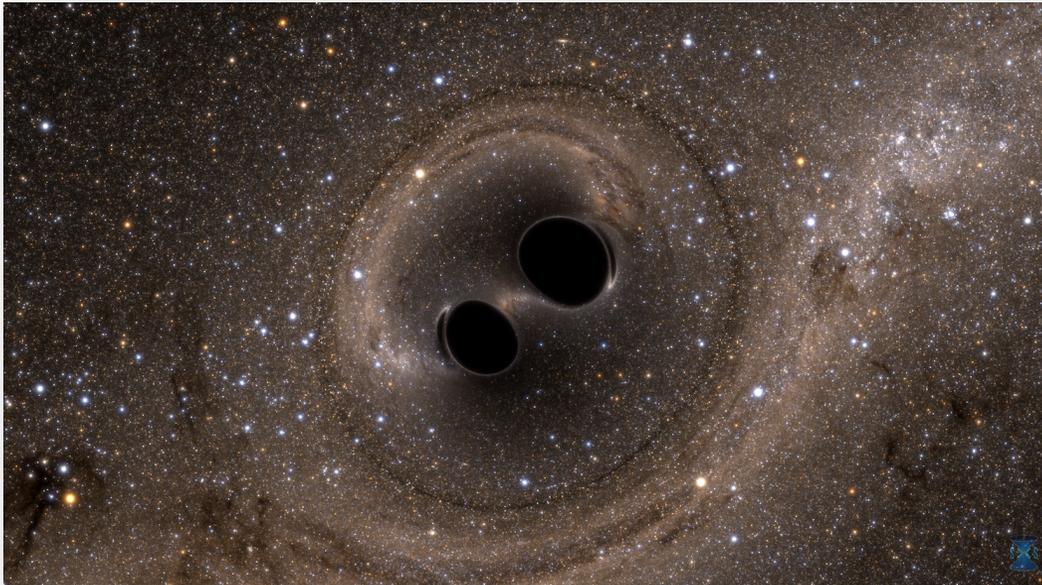


# Primordial black holes

Large densities and density fluctuations soon after the big bang.

Mass distribution should be different from that of astrophysical black holes.

Primordial BHs were suggested as Dark Matter, and the origin of some LIGO BBH mergers, e.g. GW150914 (Bird+ 2016)



# Black hole mass distribution

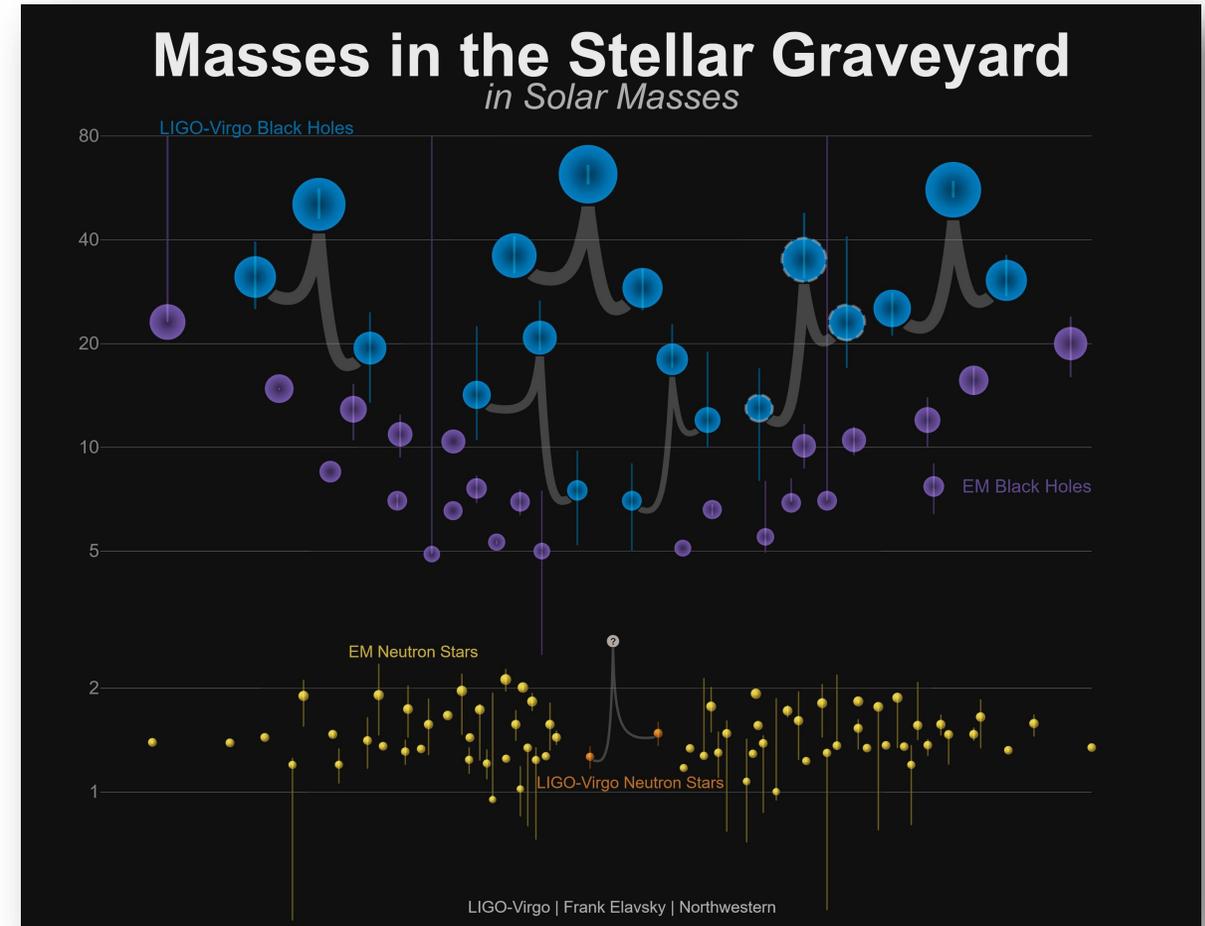
Three classes with different origin / evolution

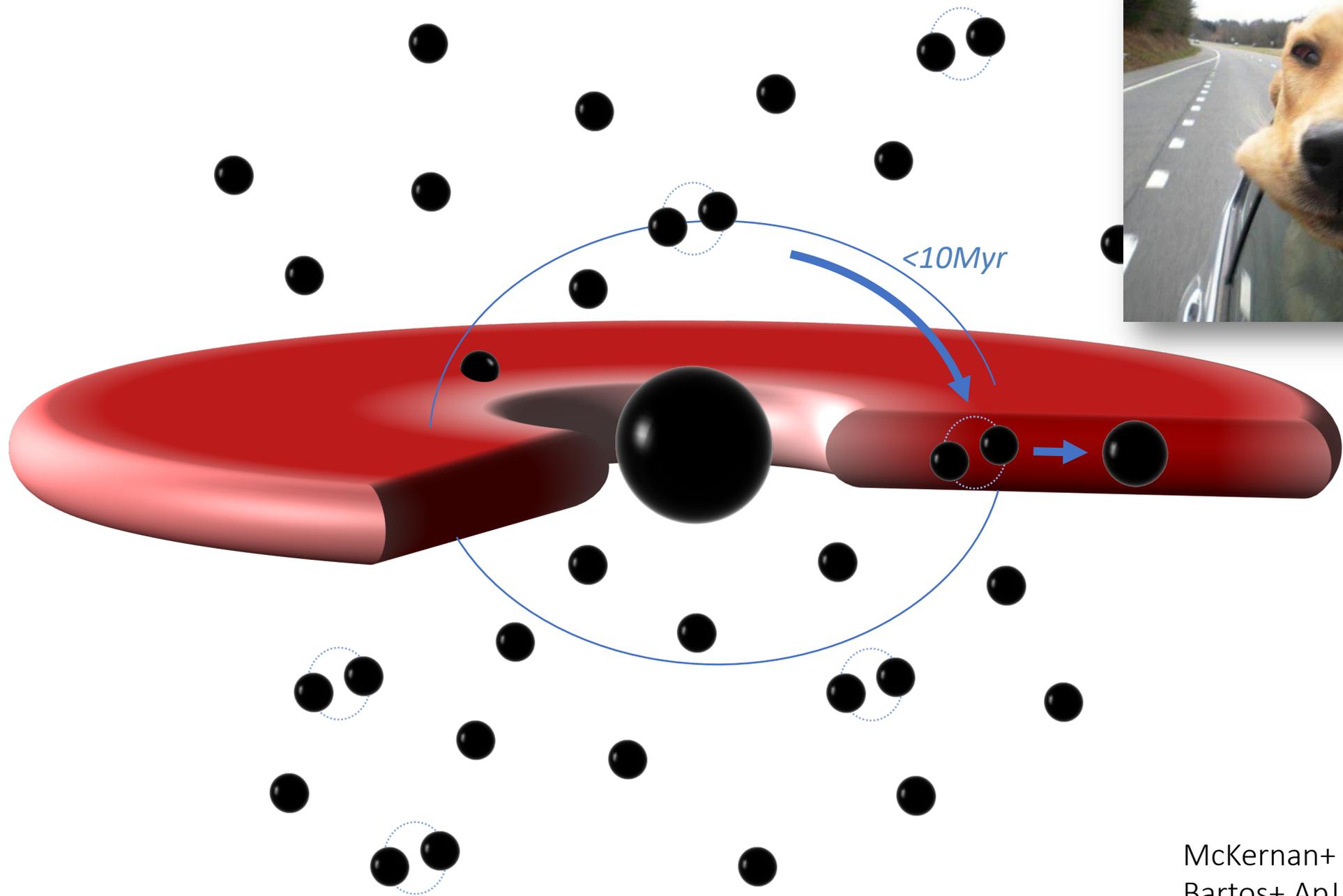
- Stellar-mass (3 Msun – ~100 Msun )
- Intermediate-mass (~100 Msun –  $10^5$  Msun)
- Supermassive ( $10^5$  Msun –  $10^{11}$  Msun)

Stellar mass mass distribution:

- We don't know
- There seems to be a mass lower limit at 5 Msun  
→ mass gap
- Best guess – Salpeter function (PDF  $\sim M^{-2.3}$ )

LIGO's detected BH mass distribution is consistent with Salpeter function up to a cutoff mass of ~ 50 Msun





# Intermediate mass black holes

No confirmed observation.

Claims:

- Measurement of Doppler shift of stellar radiation in X-ray binaries.
- Super-Eddington radiation in X-ray binaries.
- Stellar dynamics in globular clusters.
- ...

LIGO has limits on their abundance.

Origin:

- From stellar mass BHs through accretion
- Collision of multiple stars or stellar remnants in dense environments
- Primordial BHs
- Collapse of Pop III stars



# Supermassive black holes

Formation: needs a seed

- Very massive star collapses
- Primordial black hole

*But we don't know.*

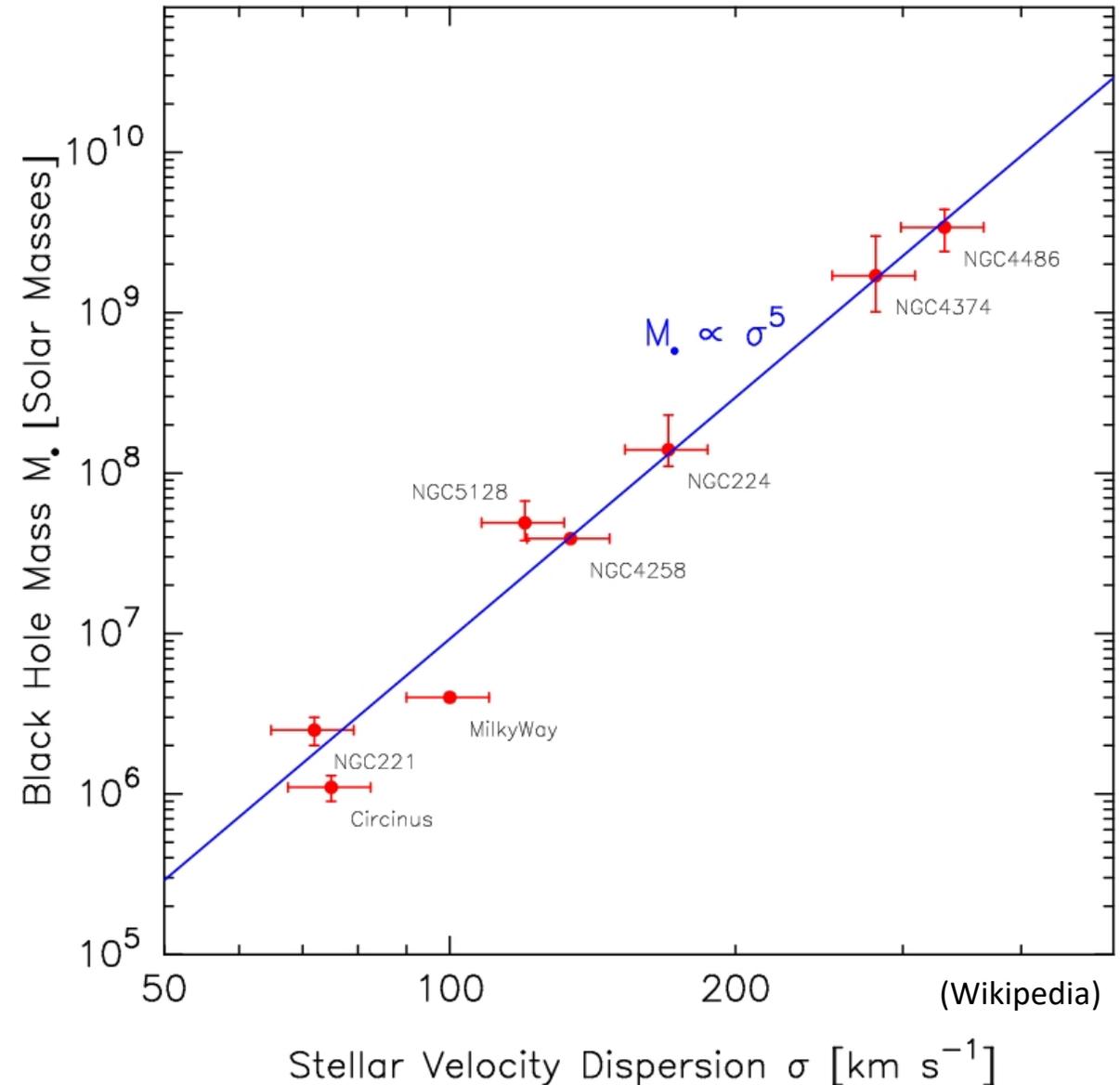
Growth:

- Accretion
- Merger with other black holes

*But we don't really know.*

They are fundamental components of galaxies:

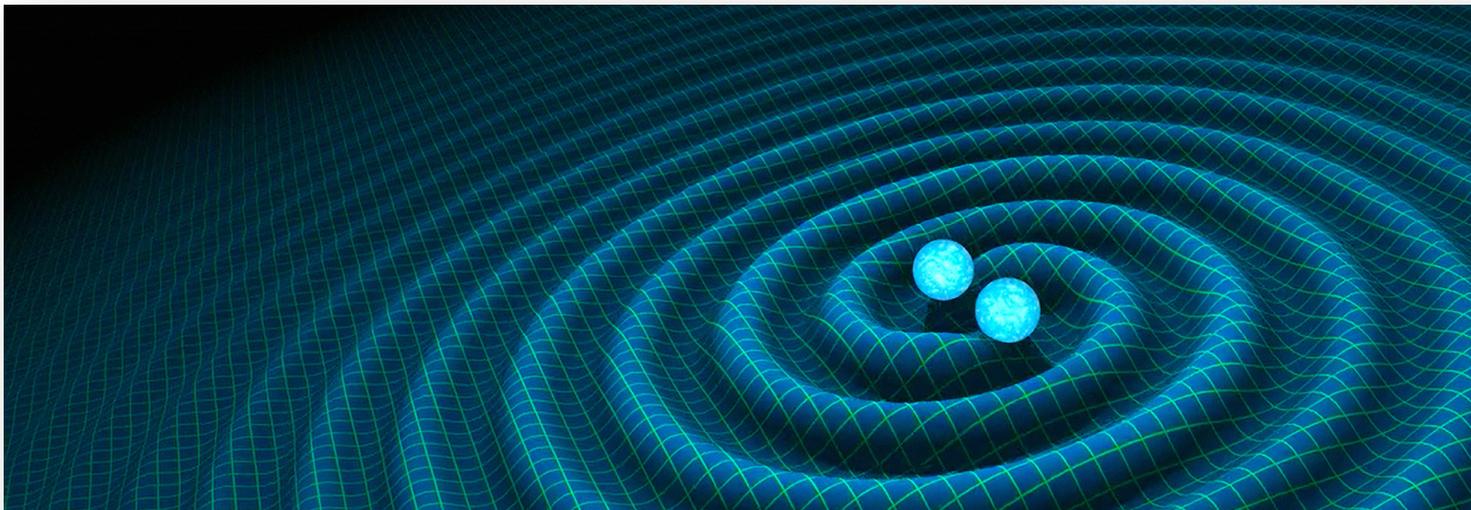
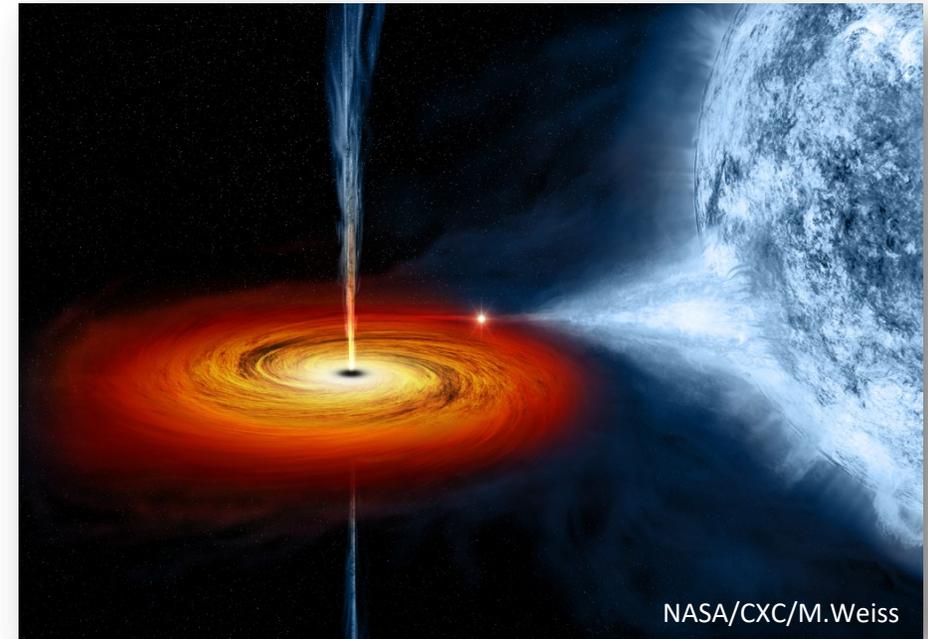
- M-sigma relation.



# Observations

Hawking radiation is undetectable.

- Gravitational waves!
- Motion of stars orbiting Sagittarius A\*
- Accretion of matter
- Gravitational lensing (not yet observed)



# Event Horizon Telescope

- Network of radio telescopes.
- Interferometry
- Observe the immediate environment of the SMBH in the Milky Way (Sagittarius A\*); and maybe others.
- Can resolve distance on the order of the Schwarzschild radius.
- Results expected soon...

