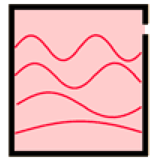
Planck Distribution

<u>Review Blackbody Radiation</u>



Set of oscillating EM modes in a cavity

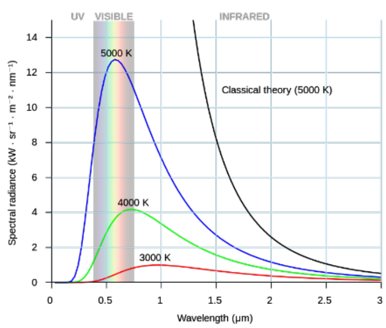
Classical. Each mode in cavity has energy $k_B T$

Density of states (unit volume) $D(k)dk = \frac{1}{2\pi} k^2 dk$

or in terms of frequency $D(v)dv = \frac{8\pi v^2}{c^3} dv$

diverges at high ν called <u>ultraviolet catastrophe</u>

Not what is observed (if true hot fireplace would emit huge number of X-rays)

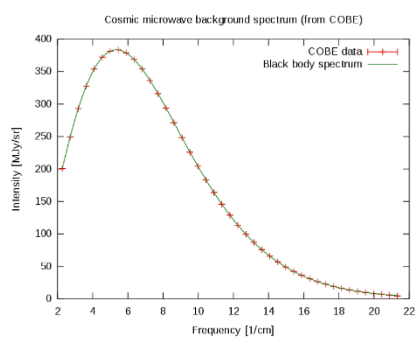


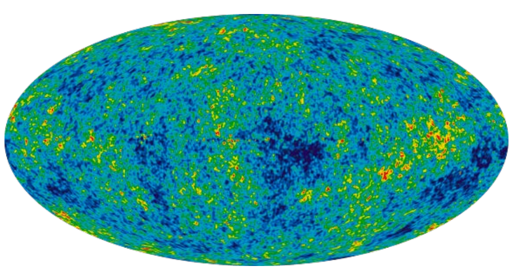
Credit: https://en.wikipedia.org/wiki/Black-body_radiation#/media/File:Black_body.svg

Cosmic background radiation (relic after Big Bang)

Fits universal curve (Planck distribution) perfectly

for T= 2.726K





https://en.wikipedia.org/wiki/Cosmic_microwave_background

Planck's Model.

EM radiation consists of quanta

Energy of photon $E = hv$ h is Planck's constant 6.623 10^{-34} J.s

Treat electrons in black body source as simple harmonic oscillators

Energy $E = \left(n + \frac{1}{2}\right)\hbar\omega(k)$ n an integer

Number of particles in state $\phi(k)$

$$n(k) = \sum_{n=0}^{\infty} n\, p_n(k)$$

Probability of occupation in state k given by previous arguments for Boltzmann distribution

$$p_n(k) = \frac{e^{-\beta n\hbar\omega(k)}}{Z(k)}$$

Partition function

$$Z(k) = 1 + e^{-\beta\hbar\omega} + e^{-2\beta\hbar\omega} + e^{-3\beta\hbar\omega} + \ \dots . = \frac{1}{1 - e^{-\beta\hbar\omega}}$$

Average number of particles

$$\bar{n} = \sum_n n\, p_n(k) = \sum_n n\, e^{-\beta n\hbar\omega(k)} \frac{1}{1 - e^{-\beta\hbar\omega}}$$

Result

$$\bar{n} = \frac{1}{e^{\beta\hbar\omega} - 1}$$ Boltzmann distribution function

Average thermal energy

$$\overline{U(k)} = \hbar\omega(k)\, \bar{n}$$

Partition function $Z = Z(k_1)Z(k_2)Z(k_3)Z(k_4) \dots . = \frac{1}{1-e^{-\beta\hbar\omega(k)}}$

Free energy $F = -k_B T ln Z_{total} = -k_B T \sum ln Z(k)$

Use density of states in k-space $D(k) = \frac{V}{2\pi^2} k^2$ per mode, two modes

$$F = -k_B T \int_0^\infty D(k) \ln(1 - e^{-\beta\hbar\omega(k)})\, dk$$

Energy $E(k) = \hbar c k$

$$F = -k_B T \int_0^\infty \frac{V}{\pi^2} k^2 \ln\left(1 - e^{-\beta\hbar c k}\right) dk$$

$$= -\frac{\hbar c V}{3\pi^2} \int_0^\infty \frac{k^3\, dk}{e^{\beta \hbar c k} - 1}$$

Find free energy $F = -\frac{\pi^2 (k_B T)^4}{45(\hbar c)^3} V$

Entropy $S = -\left(\frac{\partial F}{\partial T}\right)_V = \frac{4\pi^2 {k_B}^4 T^3}{45(\hbar c)^3} V$

Internal Energy $U = F + TS = \frac{\pi^2 (k_B T)^4}{15(\hbar c)^3} V$

Pressure $P = -\left(\frac{\partial F}{\partial T}\right)_V = \frac{\pi^2 (k_B T)^4}{45(\hbar c)^3} = \frac{U}{3V}$