Phys 4523 Mid-Term 1.

1. (a) $$\Delta S_{metal} = \int_{350}^{400} C \frac{dT}{T} = C \ln \frac{8}{7} = 0.134C \quad \text{(2pts)}$$

(b) $$\Delta S_{Bath} = -\frac{\Delta Q}{T_{Bath}} = -\frac{50C}{550} = -0.091C \quad \text{(2pts)}$$

(c) $$\Delta S_{universe} = \Delta S_{metal} + \Delta S_{Bath} \quad \text{(1pt)}$$
$$= 0.043C.$$

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2. N lattice sites (constant) $\quad$ n vacant sites

N−n occupied sites $\quad$ Internal energy $U = nE$

Number of configurations

$$W = {}^{N}C_{n} = \frac{N!}{n!\,(N-n)!} \quad \text{(2pts)}$$

Entropy (Use Stirling's Approximation)

$$S = k_B \ln W = k_B\{N\ln N - \cancel{N} - n\ln n + \cancel{n} - (N-n)\ln(N-n) - \cancel{(N-n)}\} \quad \text{(2pts)}$$
$$= k_B\{N\ln N - n\ln n - (N-n)\ln(N-n)\}$$

Relation to temperature

$$\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{\partial S}{\partial n}\left(\frac{\partial n}{\partial U}\right) = \frac{1}{E}\frac{\partial S}{\partial n} \quad \text{(1 pt)}$$

$$\frac{\partial S}{\partial n} = \frac{E}{T}$$

2. (cont.)

$$\frac{\partial S}{\partial n} = k_B \left\{ -\ln n - 1 + \ln(N-n) + 1 \right\}$$

$$= k_B \ln \left\{ \frac{N-n}{n} \right\}$$

$$E = k_B T \ln \left(\frac{N-n}{n} \right)$$

$$\frac{E}{k_B T} = \ln \left(\frac{N-n}{n} \right)$$

$$e^{E/(k_B T)} = \left(\ln \left(\frac{N}{n} \right) - 1 \right)$$

(2 pts)

$$n = \frac{N}{\exp\left(\frac{E}{k_B T}\right) + 1}$$

3. ∴ Single particle partition function

$$Z_1 = e^{-\beta E_0} + 2e^{-2\beta E_0} + e^{-3\beta E_0}$$

$$= e^{-\beta E_0}\left(1 + e^{-\beta E_0}\right)^2 \qquad (2\text{pts})$$

N particle partition function

$$Z_N = (Z_1)^N \qquad (1\text{pt})$$

Helmholtz free energy

$$F = -k_BT\ln Z_N = NE_0 - 2Nk_BT\ln\left[1 + e^{-\beta E_0}\right]$$

$$= 2NE_0 - 2Nk_BT\left\{\ln\cosh\left(\frac{E_0}{2k_BT}\right) + \ln 2\right\}. \qquad (2\text{pts})$$

Seek heat capacity

$$C_V = -T\left(\frac{\partial^2 F}{\partial T^2}\right) \qquad (1\text{pt})$$

$$\frac{\partial F}{\partial T} = -2Nk_B\left\{\ln\cosh(\ ) + \ln 2\right\}$$
$$- 2Nk_BT\left\{\tanh\left(\frac{E_0}{2k_BT}\right) * \left(\frac{-E_0}{2k_BT^2}\right)\right\}. \qquad (2\text{pts})$$

$$\frac{\partial^2 F}{\partial T^2} = \sout{-2Nk_B\tanh(\ ) * \left(-\frac{E_0}{2k_BT^2}\right)}$$
$$\sout{+ 2Nk_B\left\{\tanh(\ )\left(\frac{E_0}{2k_BT^2}\right)\right\}}$$
$$+ 2Nk_B\left(\frac{E_0}{2k_BT}\right)\frac{1}{\cosh^2(\ )}\left(\frac{-E_0}{+2k_BT^2}\right)$$

use $\frac{\partial}{\partial x}\tanh x = \frac{1}{\cosh^2 x}$

$$C_V = -T\frac{\partial^2 F}{\partial T^2} = 2Nk_B\left(\frac{E_0}{2k_BT}\right)^2\frac{1}{\cosh^2\left(\frac{E_0}{2k_BT}\right)}$$