Chapter 4.

9. $W_{vac} = \frac{N!}{M!\,(N-M)!}$ $W_{interstitials} = \frac{N!}{M!\,(N-M)!}$

$$W_{tot} = \left[\frac{N!}{M!\,(N-M)!}\right]^2 \qquad U = M\epsilon.$$

$$S = k_B \ln W = 2k_B\{N\ln N - M\ln M - (N-M)\ln(N-M)\}$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_V = \left(\frac{\partial S}{\partial M}\right)_V\left(\frac{\partial M}{\partial \epsilon}\right)_V = \frac{1}{\epsilon}\frac{\partial S}{\partial M}$$

$$\frac{\partial S}{\partial M} = 2k_B\{-\ln M - 1 + \ln(N-M) + 1\}$$

$$= 2k_B \ln\left(\frac{N-M}{M}\right) = -2k_B \ln\left(\frac{M}{N-M}\right) = -2k_B \ln\left(\frac{\alpha}{1-\alpha}\right)$$

where $\alpha = M/N$

Thus $\frac{1}{T} = -\frac{2k_B}{\epsilon}\ln\left(\frac{\alpha}{1-\alpha}\right)$ or $\frac{\alpha}{1-\alpha} = e^{-\epsilon/2k_BT}$

or $\alpha = 1/\left[1+e^{\epsilon/2k_BT}\right]$

$$M = \frac{N}{1+e^{\epsilon/2k_BT}}$$

13,

4
3
2
$n=1$
ZERO $n=0$

$$W = \frac{[(N-1)+n]!}{(N-1)!\; n!}$$

$$S = k_B \ln W$$

$$= k_B \left\{ (N-1+n)\ln(N-1+n) - (N-1)\ln(N-1) - n\ln n \right\}$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_V = \left(\frac{\partial S}{\partial n}\right)_V \left(\frac{\partial n}{\partial U}\right)_V = \frac{1}{\hbar\omega}\frac{\partial S}{\partial n}$$

$$\frac{1}{T} = \frac{k_B}{\hbar\omega}\left\{ \ln(N-1+n) - \ln n \right\}$$

$$\frac{\hbar\omega}{k_B T} = \ln\left(\frac{N-1+n}{n}\right) = + \ln\left\{\frac{N-1}{n} + 1\right\}.$$

$$\frac{N-1}{n} + \cancel{1} = e^{\hbar\omega/k_B T} - 1$$

$$n = (N-1)\left[\frac{1}{e^{\hbar\omega/k_B T} - 1}\right]$$

$$\sim U = (N-1)\hbar\omega\left[\frac{1}{e^{\hbar\omega/k_B T} - 1}\right]$$

Chapter 5.

16. One particle

$$Z_1 = \sum_i e^{-\beta \epsilon_i}$$

$$= 1 + 3e^{-\beta\epsilon} + 3e^{-2\beta\epsilon} + e^{-3\beta\epsilon}$$

$$= \left(1 + e^{-\beta\epsilon}\right)^3$$

N particles

$$Z_N = \left(Z_1\right)^N = \left(1 + e^{-3\beta\epsilon}\right)^{3N}$$

$$F = -k_BT \ln Z_N = -3Nk_BT \ln\left(1 + e^{-\epsilon/k_BT}\right)$$

17.

$$Z = \left(\frac{V-Nb}{N}\right)^N \left(\frac{mk_BT}{2\pi\hbar^2}\right)^{\frac{3N}{2}} e^{N^2a^2/(Vk_BT)}$$

$$-F = k_BT \left\{ N\ln\left(\frac{V-Nb}{N}\right) + \frac{3N}{2}\left[\ln T + \ln \frac{mk_B}{2\pi\hbar^2}\right] + \frac{N^2a^2}{Vk_BT} \right\}$$

$$P = -\left(\frac{\partial F}{\partial V}\right)_T$$

$$= k_BTN\left(\frac{1}{V-Nb}\right) - \frac{N^2a^2}{V^2}$$

or $$\boxed{\left(P + \frac{N^2a^2}{V^2}\right)\left(V - Nb\right) = Nk_BT}$$

contd.

$$U = k_B T^2 \left(\frac{\partial \ln Z}{\partial T} \right)$$

$$= k_B T^2 \left\{ \frac{3N}{2} \cdot \frac{1}{T} - \frac{N^2 a^2}{V k_B T^2} \right\}$$

$$U = \frac{3}{2} N k_B T - \frac{N^2 a^2}{V}$$

$$\longrightarrow$$

Chapter 5.

21 $$U = -\frac{\partial}{\partial\beta}\ln Z$$

$$Z = \sum_i e^{-\beta\epsilon_i}$$

$$\frac{\partial}{\partial\beta}\ln Z = \frac{-\sum_i e^{-\beta\epsilon_i}\cdot\epsilon_i}{Z} = -\sum_i p_i\epsilon_i = \underline{-U}$$

Harmonic Oscillator

$$E_n = \left(n+\tfrac{1}{2}\right)\hbar\omega$$

$$Z = \sum_n e^{-\beta E_n}$$

$$= e^{-\beta\hbar\omega/2}\sum_{n=0}^{\infty} e^{-\beta n\hbar\omega}$$

$$= e^{-\beta\hbar\omega/2}\cdot\frac{1}{1-e^{-\beta\hbar\omega}}$$

$$\ln Z = -\beta\frac{\hbar\omega}{2} - \ln\left(1-e^{-\beta\hbar\omega}\right)$$

$$\frac{\partial\ln Z}{\partial\beta} = -\frac{\hbar\omega}{2} - \frac{\hbar\omega}{e^{\beta\hbar\omega}-1}$$

$$U = \frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\beta\hbar\omega}-1}$$