Homework 1

Chapter 1

1. Van der Waal's equation is
   $(P + \frac{a}{V^2})(V - b) = \theta R.$
   We seek an equation of state of the form $P = G(\theta, V)$
   Since $\left(P + \frac{a}{V^2}\right) = \frac{\theta R}{V-b}$, we find $P = -\frac{a}{V^2} + \frac{\theta R}{V-b}$

2. $U(T, V)$ is a function of state

   This in $dQ = dU + PdV$ we can write

$$dQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial T}\right)_V dV + PdV = \left(\frac{\partial U}{\partial T}\right)_V dT + \left[P + \left(\frac{\partial U}{\partial T}\right)_V\right] dV$$

4a

Try $y^\alpha z^\beta$ as an integrating factor for $12z^2$dy +18yzdz

$$\frac{d}{dz}\left(y^\alpha z^\beta 12z^2\right) = 12(2+\beta)y^\alpha z^{1+\beta}$$

$$\frac{d}{dy}\left(y^\alpha z^\beta 18yz\right) = 18(1+\alpha)z^{1+\beta}y^\alpha$$

Have an exact differential if $\beta = \frac{1}{2}(3\alpha - 1)$

4b

$\frac{d}{dz}\left(y^\alpha z^\beta 2e^{-z} = 2y^\alpha\left[\beta z^{\beta-1}e^{-z} + z^\beta(-1)e^{-z}\right]\right)$ and

$$\frac{d}{dy}\left(y^\alpha z^\beta(-ye^{-z})\right) == -z^\beta e^{-z}[(1+\alpha)y^\alpha]$$

To be an exact differential we would require $2[\beta z^{-1} - 1] = -(1+\alpha)$

There is no solution for this form of an integrating factor.

6. $dQ = (dQ)_V + (dQ)_T = C_V(dT)_V + C_P(dT)_P$

From the ideal gas law

$(dT)_V = VdP/nR$ and $(dT)_P = PdV/nR$

whence $dQ = (C_V VdP + C_P PdV)$/nR

7. $P = aV^{-\gamma}$

Work for specified path is

$$W = -\int_{V_1}^{V_2} PdV = \frac{a}{\gamma - 1}\left[V_2^{1-\gamma} - V_1^{1-\gamma}\right]$$