Homework 5 Phy 4523

8.6 Debye model

Start with Eq'n 8.8.15

Free energy

$$F = N\epsilon + \frac{3k_BTV}{2\pi^2 \bar{s}^3}\int_0^{\omega_D} \omega^2 \ln\left(1-e^{-\beta\hbar\omega}\right) d\omega$$

$$\frac{\partial F}{\partial T} = \frac{3k_BV}{2\pi^2\bar{s}^3}\int_0^{\omega_D} \omega^2 \ln\left(1-e^{-\beta\hbar\omega}\right) + \frac{3V}{2\pi^2\bar{s}^3}\int_0^{\omega_D} \frac{-\hbar\omega^3}{T}\frac{d\omega}{(e^{\beta\hbar\omega}-1)}$$

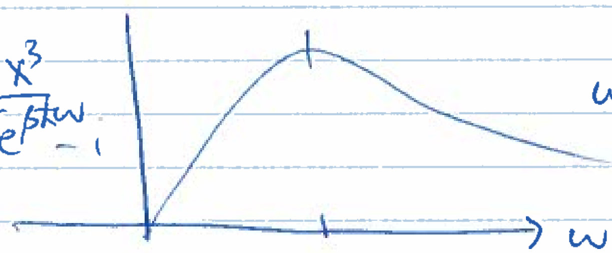
$$U = F+TS = F - T\left(\frac{\partial F}{\partial T}\right)_V$$

$$= N\epsilon + \int_0^{\omega} \frac{3V\omega^2}{2\pi^2\bar{s}^3}\frac{\hbar\omega}{(e^{\beta\hbar\omega}-1)} d\omega$$

$$= N\epsilon + \frac{3V}{2\pi^2\bar{s}^3}\int \frac{\hbar\omega^3}{e^{\beta\hbar\omega}-1} d\omega$$

Integrand

$$y \sim \frac{x^3}{e^{\beta\hbar\omega}-1}$$



$$\omega_{max} \sim \frac{2k_BT}{\hbar}$$

$$x_{max} \sim 2.9$$

$$y \sim \frac{x^3}{e^{x}-1}$$

At 30K $\quad E = 2.7 * k_B * 30 \quad \sim 10^{-21}$ J

600K $\quad \sim 2*10^{-20}$ J.

8.12 $S = k_B c^3 A / 4 G\hbar$

$A = 4\pi R_s^2 \qquad R_s = 2Gm/c^2$

Thus $S = \dfrac{k_B 4\pi G M}{\hbar c^5} U^2 \qquad U = mc^2$

$\dfrac{1}{T} = \dfrac{\partial S}{\partial U} = \dfrac{8 k_B \pi}{\hbar c^5}$

$T = \dfrac{\hbar c^5}{8 k_B \pi} \qquad \dfrac{\partial U}{\partial t} = c^2 \dfrac{\partial m}{\partial t}$

$\dfrac{\partial U}{\partial t} = c^2 \dfrac{\partial m}{\partial t} = -A\sigma T^4 = -4\pi R_s^2 \sigma T^4.$

$= -16\pi \dfrac{G^2 M^2}{c^4} \sigma \left(\dfrac{\hbar c^3}{8 k_B \pi\, G M}\right)^4 = c^2 \dfrac{dM}{dt}$

$dt = \dfrac{256 \pi^3 G^2 k_B^4}{\sigma c^6} M^2 dM$

$t = -\dfrac{256}{3} \dfrac{\pi^3 G^2 k_B^4}{\sigma c^6} (\Delta M)^3$

$\Delta M \simeq 2 \times 10^{11}\,\text{kg} \qquad t \sim 6 \times 10^{17}\,\text{s}$

~ 20 billion yrs

9.2

$$\hat{H}|\psi_i\rangle = E_i|\psi_i\rangle$$

$$\hat{L}_z|\psi_i\rangle = L_{z_i}|\psi_i\rangle$$

Define $\omega_z = -T\left(\frac{\partial S_R}{\partial L_z}\right)_R$

$$\frac{1}{T} = \frac{\partial S_R}{\partial U_R} = k_B \frac{\partial \ln W_R}{\partial U_R}$$

$$\omega_z = -T\frac{\partial S_R}{\partial L_z} = -k_B T\frac{\partial \ln W_R}{\partial L_z}$$

$$W_R = K \exp\left(\frac{U_R - \omega_z L_{zR}}{k_B T}\right)$$

$U_R = U_T - E_i$

$L_{z_R} = L_{z_T} - L_{z_i}$

$$W_i \propto \exp\left[-(E_i - \omega_z L_{z_i})/k_B T\right]$$

Probability

$$p_i = \frac{e^{-\beta(E_i - \omega_z L_{z_i})}}{\sum_i e^{-\beta(E_i - \omega_z L_{z_i})}}$$

Let $\Pi = \sum_i e^{-\beta(E_i - \omega_z L_{z_i})}$ analog of grand partition fn

Grand potential.

$$\Phi_G = -k_B T \ln \Pi = \bar{U} - ST - \omega_z \bar{L}_z$$

9.8 Φ_G = grand potential

$$d\Phi_G = -PdV - Nd\mu - SdT$$

$$\frac{d\Phi_G}{dV} = -P\Big)_{N,T} = \frac{\Phi_G}{V}$$ extensive variable

Use

$$d\Phi_G = -PdV - VdP$$

Thus $-PdV - VdP = -PdV - Nd\mu - SdT$

or $\boxed{VdP = Nd\mu + SdT}$

9.9. M sites, N adher, M-N atoms free

$$W = \frac{M!}{N!\,(M-N)!}$$

Grand partition function

$$Z = \sum_N \frac{M!}{N!\,(M-N)!} e^{-\beta(U-\mu N)} = \sum \cdots e^{-\beta N(E-\mu)}$$

$$= \left(1 + e^{-\beta(E-\mu)}\right)^M$$

9.8 Φ_G = grand potential

$$d\Phi_G = -PdV - N d\mu - SdT$$

$$\frac{d\Phi_G}{dV} = -P\Big)_{N,T} = \frac{\Phi_G}{V}$$ extensive variable

Use
$$d\Phi_G = -PdV - VdP$$

Thus $-PdV - VdP = -PdV - Nd\mu - SdT$

or $\boxed{VdP = Nd\mu + SdT}$

9.9. M sites, N adher, M−N atoms free

$$W = \frac{M!}{N!\,(M-N)!}$$

Grand partition function

$$\mathcal{Z} = \sum_{N} \frac{M!}{N!\,(M-N)!} e^{-\beta(U-\mu N)} = \sum \cdots e^{-\beta N(E-\mu)}$$

$$= \left(1 + e^{-\beta(E-\mu)}\right)^M$$