Phy 4523

Homework #4

Chap 6. #2

$$Z_1 = e^{-3\beta\epsilon/2} + 3e^{-\beta\epsilon/2} + 3e^{\beta\epsilon/2} + e^{3\beta\epsilon/2}$$

$$= e^{-3\beta\epsilon/2}\left[1 + 3e^{\beta\epsilon} + 3e^{2\beta\epsilon} + e^{3\beta\epsilon}\right]$$

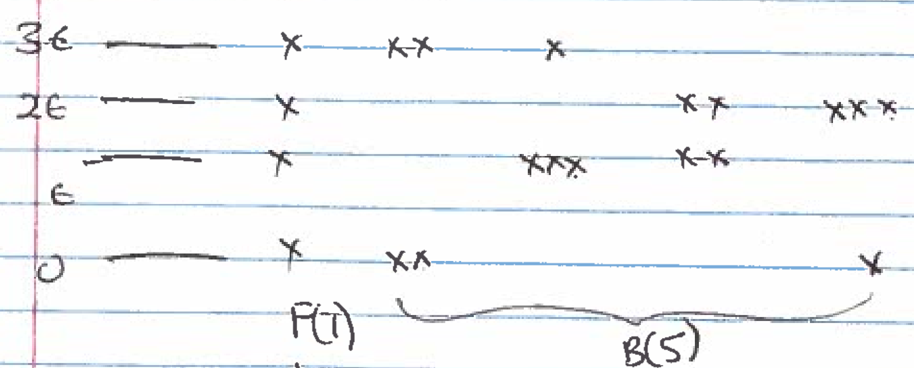
$$= e^{-3\beta\epsilon/2}\left(1 + e^{\beta\epsilon}\right)^3$$

$$= \left(e^{-\beta\epsilon/2} + e^{\beta\epsilon/2}\right)^3 = 2^3 \cosh\left(\frac{\beta\epsilon}{2}\right)$$

$$Z_N = (Z_1)^N = 2^{3N} \cosh^N\left(\frac{\beta\epsilon}{2}\right)$$

$$F = -k_BT \ln Z_N = -3Nk_BT \ln 2 + (-Nk_BT) \ln \cosh\left(\frac{\beta\epsilon}{2}\right)$$

#4. # Ways to distribute 6ϵ among 4 non-degenerate particle states with energy $0, \epsilon, 2\epsilon, 3\epsilon$



$4! = 24$ $\quad {}_4C_2 = 6$ $\quad 4$ $\quad {}_4C_2$ $\quad 4$ $\qquad \Sigma = 44$

Chap 6

#6 No of configurations

$$W = {}^{N}C_{n} \cdot 3^{n} = \frac{N!}{n!\,(N-n)!}\,3^{n}$$

arrange spins

Entropy

$$S = k_B \ln W = n\ln 3 + N\ln N - \cancel{N} - n\ln n + n$$
$$- (N-n)\ln(N-n) + (N-n)$$

$$= N\left\{\frac{n}{N}\ln 3 + \ln N - \frac{n}{N}\ln N - \left(1-\frac{n}{N}\right)\ln(N-n)\right\}$$

$$= N\left\{x\ln 3 - (1-x)\ln(1-x) - x\ln x\right\}$$

where $x = \frac{n}{N}$

$$S = \frac{\partial U}{\partial T} = \frac{\partial U}{\partial n}\left(\frac{\partial n}{\partial T}\right) \qquad U = Nx\epsilon$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_V = \left(\frac{\partial S}{\partial x}\right)\left(\frac{\partial x}{\partial U}\right)$$

$$\frac{\partial S}{\partial x} = k_B N\left\{\ln 3 - \ln x + \ln(1-x)\right\}$$

$$\frac{1}{T} = \frac{1}{\epsilon}k_B \ln\left\{\frac{3(1-x)}{x}\right\}$$

$$\frac{3}{x} - 3 = e^{\epsilon/k_BT}$$

$$\text{or } x = \boxed{\frac{n}{N} = \frac{3}{3+e^{\beta\epsilon}}}$$

17

#7 Relativistic particle, energy $E = \hbar c k$
$(p \gg mc)$

$$k = \frac{E}{\hbar c}$$

Density of states in 1D

$$D(E) = \frac{L}{\pi}\left(\frac{dk}{dE(k)}\right) dE \qquad \frac{dk}{dE} = \frac{1}{\hbar c}$$

$$= \left(\frac{L}{\pi \hbar c}\right) dE$$

Partition function

$$Z = \int_0^\infty D(E) e^{-\beta E} dE = \frac{L}{\pi\hbar c}\int e^{-\beta E} (d\beta E)\, k_B T$$

$$= \frac{L k_B T}{\pi \hbar c}$$

#17. Consider 3D density of states

$$D(k) = \frac{V}{2\pi^2} k^2 \qquad p = mu = \hbar k$$

$$k = \frac{mu}{\hbar}$$

Number density

$$n(u) = \frac{V}{2\pi^2} \frac{k^2}{Z} e^{-\beta m u^2}$$

$$= \frac{V}{2\pi^2}\left(\frac{m}{\hbar}\right)^3 \frac{u^2}{Z} e^{-\frac{mu^2}{2}\beta}$$

7.17.

Distribution of speeds at hole

$$u\,n(u) = \frac{V}{2\pi^2}\left(\frac{m}{h}\right)^3 \frac{1}{2}\, u^3 e^{-\frac{1}{2}\beta m u^2}$$

Average kinetic energy of particles leaving

$$\langle \tfrac{1}{2}mu^2 \rangle = \frac{\int (\frac{1}{2}mu^2)\, u n(u)\, du}{\int u n(u)\, du}$$

$$= \frac{m}{2} \frac{\int u^5 e^{-\frac{1}{2}\beta m u^2} du}{\int u^3 e^{-\frac{1}{2}\beta m u^2} du.}$$

$$= \frac{m}{2} \cdot \left(\frac{2k_BT}{m}\right) \int x^5 e^{-x^2} dx \Big/ \int x^3 e^{-x^2} dx$$

$$= \underrightarrow{2k_BT}$$