▷ Conductors.

↪ • Materials in which charges can move freely.

- electrons in metallic conductors
- ions in ionic conductor such as salt water.

• an ideal conductor or perfect conductor has 0 resistivity.

$$\rho_r = 0 \quad \text{or} \quad \sigma \to \infty \quad \left(\rho_r = \frac{1}{\sigma} \right)$$

resistivity $\quad$ conductivity

Here, our discussion is on an ideal conductor.

(i) $\vec{E} = 0$ inside a conductor : No force on charges

(ii) No ~~excess~~ net charges inside a conductor : $\rho = 0$

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \quad (\vec{E} = 0 \text{ inside})$$

(iii) Any net charges resides on the surface.

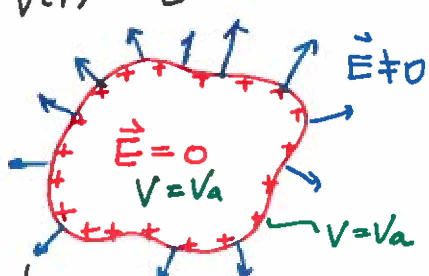
(iv) A conductor is an equipotential

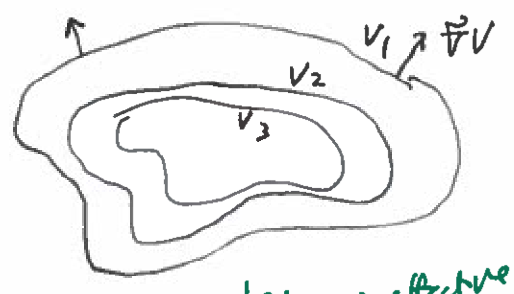
$$V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{r} = 0 \quad \text{inside a conductor.}$$

⇒ Surface of a conductor is an equipotential surface

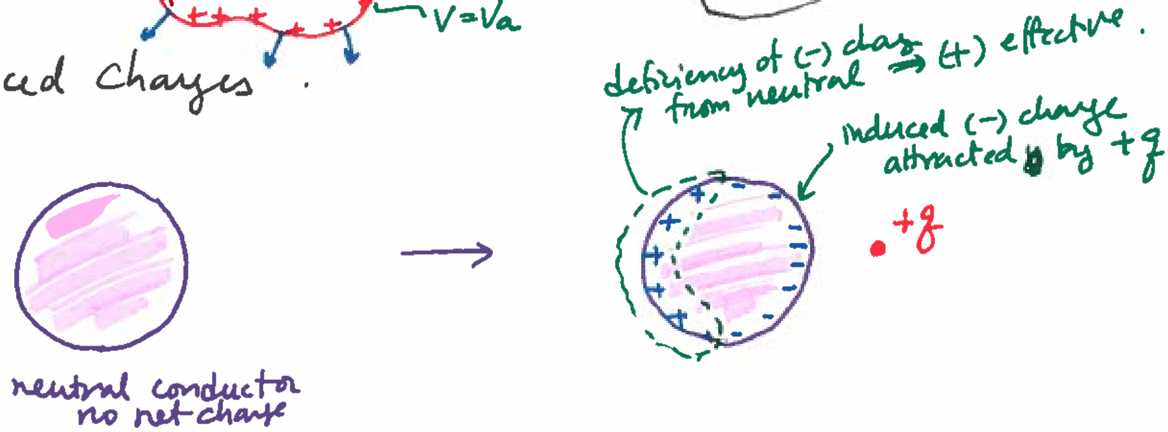
(v) Therefore, $\vec{E}$ on the surface is perpendicular to the surface.

$$-\vec{\nabla} V(\vec{r}) = \vec{E}$$

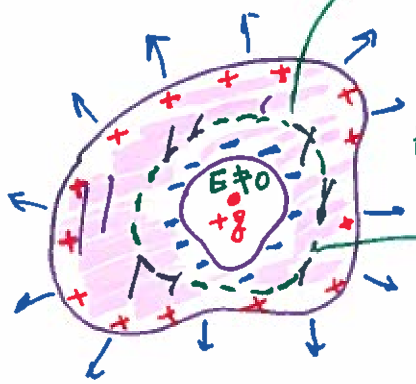




▷ Induced charges.



Inside $\vec{E}=0$



$\vec{E}\neq 0$

Gaussian surface

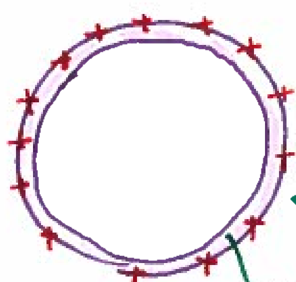
Inside $q_{en} = +q + \boxed{q_{ind}} = 0$

$= -q$

In a perfect conductor, the induced charge completely cancel out the presence of $+q$ in the cavity

Ex

(1)

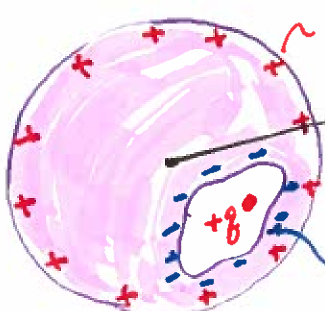


total net charge on a conducting shell $= +q$

only on the outer surface.

$\vec{E}=0$ inside the shell. (no charge on the inner surface)

(2)



$+q$

r

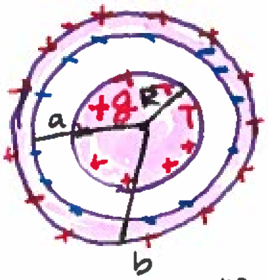
$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{+q}{r}}$$

$-q$ (completely screen $-q$ out).

neutral conducting sphere w/ cavity.

$\boxed{HW}$ 2.38 & 2.39

2.38



$$\sigma_R = \frac{+q}{4\pi R^2} \; ; \; \sigma_a = \frac{-q}{4\pi a^2} \; ; \; \sigma_b = \frac{+q}{4\pi b^2}$$

$$\vec{E} = E\hat{r} \quad \text{where} \quad E(r) = \begin{cases} 0 & (r<R) \\ \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} & (R<r<a) \\ 0 & (a<R<b) \\ \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} & (r>b). \end{cases}$$

$$V(0) = -\int_{\infty}^{r=0} \vec{E}\cdot d\vec{r} = -\int_{\infty}^{b} \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{r^2}dr + \int_b^a 0\,dr + \int_a^R \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}dr + \int_R^0 0\,dr$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} + \frac{1}{R} - \frac{1}{a}\right)$$

If you connect the outer shell to GND, then the charge (+) will be drained to GND. $\sigma_b \to 0$, consequently $E(r)=0$ for $r>a$.

$$\therefore\; V(0) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{a}\right).$$

- Surface charge and the force on a conductor.

Boundary condition.

$$\underbrace{\vec{E}_{out}}_{\text{outside conductor}} - \underbrace{\vec{E}_{in}}_{\text{inside conductor}} = \frac{\sigma}{\varepsilon_0}\hat{n} \quad \Rightarrow \text{"Discontinuity in } \vec{E} \text{ at the surface charge"}$$

$\vec{E}_{in} = 0$.

$$\vec{E}_{out} = \vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

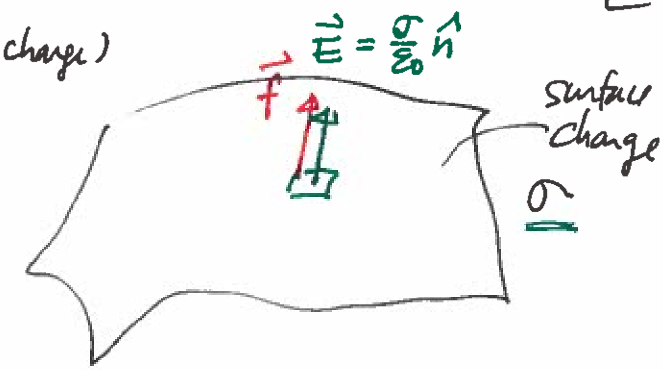
or
$$\frac{\partial V_{out}}{\partial n} - \underbrace{\frac{\partial V_{in}}{\partial n}}_{=0\text{ (equipotential)}} = -\frac{1}{\varepsilon_0}\sigma \qquad \Big(\underbrace{\frac{\partial V}{\partial n}}_{\text{normal derivative}} = \vec{\nabla}V\cdot\hat{n}\Big).$$

$$\therefore\; \frac{\partial V_{out}}{\partial n} = \frac{\partial V}{\partial n} = -\frac{1}{\varepsilon_0}\sigma.$$

Presence of $\vec{E}$ where σ (surface charge)

$\Rightarrow$ force on σ

$\vec{f}$: force on the surface charge of unit area. $\therefore q = \sigma(1) = \sigma$.



$\therefore \vec{f} = \sigma \vec{E}$ → What is the proper $\vec{E}$ here?

$$\vec{E} = \vec{E}_{avg} = \frac{1}{2}(\vec{E}_{out} + \vec{E}_{in}) = \frac{1}{2}\frac{\sigma}{\varepsilon_0}\hat{n}$$

$$\therefore \vec{f} = \frac{1}{2}\frac{\sigma^2}{\varepsilon_0}\hat{n}.$$

$$\therefore d\vec{f} = \frac{1}{2}\frac{\sigma^2}{\varepsilon_0} da\,\hat{n} = \frac{1}{2}\frac{\sigma^2}{\varepsilon_0} d\vec{a}$$

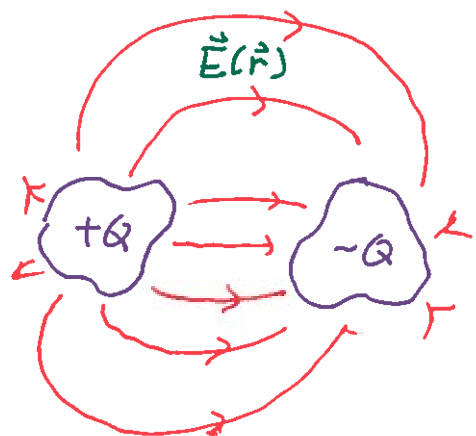
For an ~~ab~~ arbitrary surface of a conductor, the total force

$$\vec{F} = \int_S d\vec{f} = \frac{1}{2}\int_S \frac{\sigma^2}{\varepsilon_0} d\vec{a} \qquad (\sigma \text{ could be non-uniform.})$$

▸ Capacitor (more than one conductor).

The electric pot. difference btwn two conductors

$$\Delta V = V_+ - V_- = \underbrace{-\int_{(-)}^{(+)} \vec{E}\cdot d\vec{r}}_{\text{depends on the exact distribution of surface charges on the conductors}}$$



depends on the exact distribution of surface charges on the conductors

⇓

geometrical effect!

But we know $|\vec{E}| \propto Q$.

$\therefore \Delta V \propto Q \cdot \boxed{}$ — geometrically determined factor.

$\boxed{\ } \equiv \frac{1}{C}$ where C is called capacitance.

$\therefore \quad \Delta V = \frac{Q}{C}$ or $\underline{Q = C\Delta V}$.

for a given ΔV, the amount of charge on the conductor is ~~determ~~ proportional to C (capacitance).

$[C] = [\frac{Q}{V}] = \frac{C}{V} = F$ (farad).

- What is the sign of ΔV?
  Always $\Delta V > 0$ $\therefore$

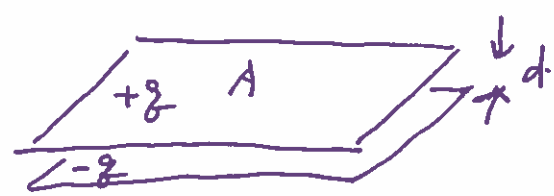
- How to calculate a capacitance?

(i) Assign $+q$ & $-q$ on each conductor.

(ii) Calculate $\vec{E}$ and $\Delta V = V_+ - V_-$.

(iii) Then you'll establish $\Delta V = \underbrace{\boxed{\ }}_{\frac{1}{C}}\, q$

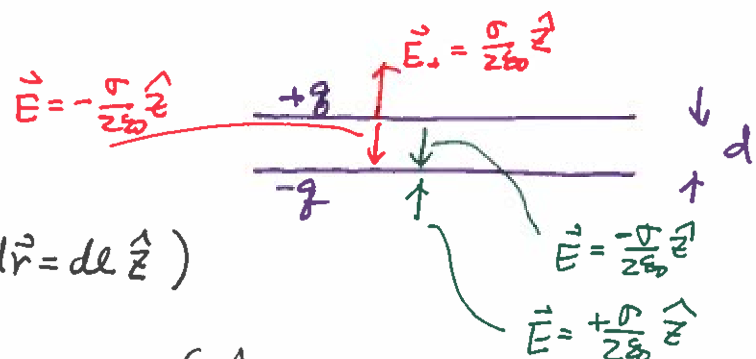
EX (1) Parallel plate capacitor



Between the plates

$$\vec{E} = -\frac{\sigma}{\varepsilon_0}\hat{z} \quad \left(\sigma = \frac{q}{A}\right)$$

$$= -\frac{q}{\varepsilon_0 A}\hat{z}$$



$$\therefore \Delta V = V_+ - V_-$$

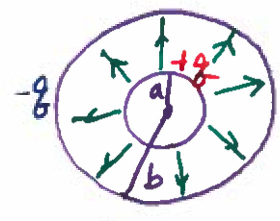
$$= -\int_{(-)}^{(+)} \vec{E}\cdot d\vec{r} \quad (d\vec{r} = dl\,\hat{z})$$

$$= \frac{d}{\varepsilon_0 A}\cdot q \qquad \therefore \quad C = \frac{\varepsilon_0 A}{d}.$$

(2) Concentric spherical shells.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \quad \text{(between the shell)}.$$

$$\therefore \Delta V = V_+ - V_- = -\int_{(-)}^{(+)} \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r}\cdot d\vec{r}$$



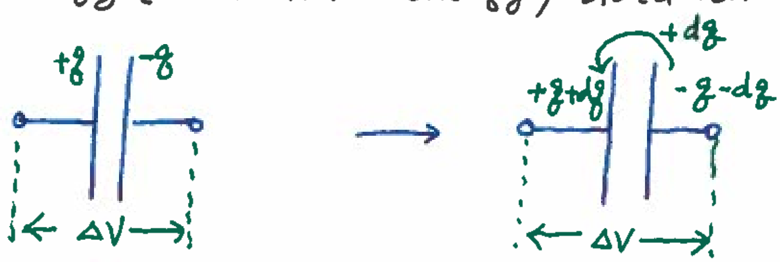
$$= \frac{-q}{4\pi\varepsilon_0}\int_b^a \frac{1}{r^2}dr \qquad (d\vec{r} = dr\,\hat{r})$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r}\right)\Big|_b^a = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$\therefore C = 4\pi\varepsilon_0\cdot\frac{ab}{b-a}. \quad (>0).$$

$\boxed{\text{HW}}$ 2.43

◎ Energy (electrostatic energy) stored in a capacitor.



Capacitance: C

$$\Delta V = \frac{q}{C}$$

The amount of work to move $+dq$ from (−) plate to (+) plate for an arbitrary charge q

$$dW = \Delta V\,dq = \frac{q}{C}dq$$

∴ total amount of work to establish charge q from 0 (nothing)

$$W = \int_0^q \frac{q'}{C}dq' = \frac{1}{2}\frac{q^2}{C} \quad \text{or}$$

$$= \frac{1}{2}C(\Delta V)^2$$

$\boxed{\text{HW}}$ 2.44 and 2.54 (additional)

[W] = [E] [Force] = [Pressure]·[Area] ∴ [pressure] = $\frac{[\text{Energy}]}{[\text{Volume}]}$