$\boxed{II.7.}$ CH.5. Magnetostatics.

▸ Lorentz Force on a moving charge in $\vec{B}$

$$\vec{F}_q = q(\vec{v} \times \vec{B}).$$

$\vec{v} \longrightarrow$ velocity dependent force, unique!

$$[B] = \frac{[F]}{[q][v]} = \frac{N}{C\cdot(m/s)} = \frac{N\cdot S}{C\cdot m} = T \text{ (tesla)}.$$

or $\vec{F}_q = q\vec{E}$: force on a charge in $\vec{E}$.

$$\therefore [B] = \frac{[E]}{[v]} = \frac{(N/C)}{(m/s)} = \frac{N\cdot S}{C\cdot m}.$$

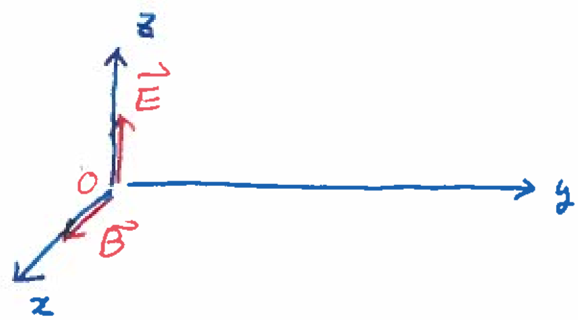
$$\therefore \vec{F}_q = q(\vec{E} + \vec{v}\times\vec{B}).$$

$\overline{\underline{EX}}$ Cycloid Motion.

A positive charge is released at O.

$\vec{v}(0) = 0, \quad \vec{r}(0) = 0.$

$\vec{E} = E_0\hat{z}$ and $\vec{B} = B_0\hat{x}$.



$$\therefore \vec{F}_q = q(\vec{E} + \vec{v}\times\vec{B}).$$

$$\vec{v}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ B_0 & 0 & 0 \end{vmatrix} = B_0 v_z \hat{y} - B_0 v_y \hat{z}.$$

$$\therefore \vec{F}_q = q(E_0\hat{z} + B_0 v_z\hat{y} - B_0 v_y\hat{z}) = qB_0 v_z\hat{y} + q(E_0 - v_y B_0)\hat{z}.$$

$$m\ddot{y} = qB_0\dot{z} \quad \text{and} \quad m\ddot{z} = q(E_0 - B_0\dot{y}).$$

Define cyclotron frequency $\omega \equiv \frac{qB_0}{m}$ (check if ω has dimension of t^{-1}).

$$\left.\begin{aligned} \ddot{y} &= \omega\dot{z} \\ \ddot{z} &= \omega\left(\frac{E_0}{B_0} - \dot{y}\right). \end{aligned}\right\} \; (*)$$

$\dddot{y} = \omega \ddot{z} \longrightarrow$ into (*).

$\dddot{y} = -\omega^2(\dot{y} - \frac{E_0}{B_0})$ ——— ⑥

By substituting $\dot{y} - \frac{E_0}{B_0} \equiv u$

Eq. ⑥ becomes. $\ddot{u} + \omega^2 u = 0$. Then you can find.

$$y(t) = C_1 \cos\omega t + C_2 \sin\omega t + \left(\frac{E_0}{B_0}\right)t + C_3$$

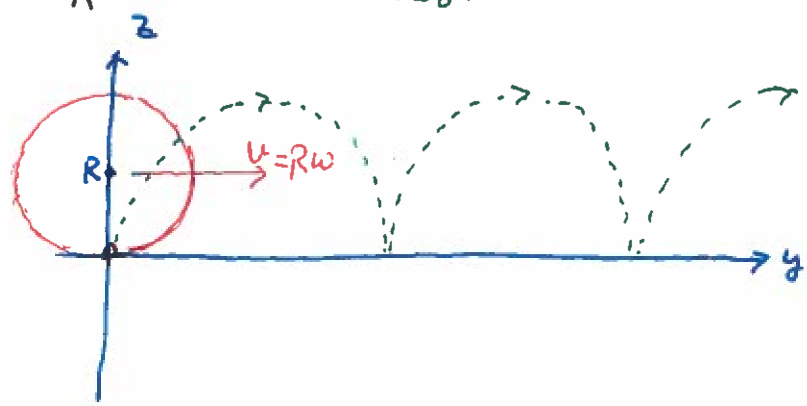
$$z(t) = C_2 \cos\omega t - C_1 \sin\omega t + C_4.$$

Initial conditions $\vec{v}(0) = \vec{r}(0) = 0$.

$$\left.\begin{aligned} y(0) &= C_1 + C_3 = 0 && \Rightarrow C_3 = -C_1 \\ z(0) &= +C_2 + C_4 = 0 && \Rightarrow C_4 = -C_2 \\ \dot{y}(0) &= +\omega C_2 + \frac{E_0}{B_0} = 0 \\ \dot{z}(0) &= -\omega C_1 = 0 \end{aligned}\right\} \quad \begin{aligned} C_1 &= C_3 = 0 \\ C_2 &= -\frac{E_0}{\omega B_0} \\ C_4 &= \frac{E_0}{\omega B_0}. \end{aligned}$$

$$\therefore \left.\begin{aligned} y(t) &= \frac{E_0}{\omega B_0}(\omega t - \sin\omega t) \\ z(t) &= \frac{E_0}{\omega B_0}(1 - \cos\omega t). \end{aligned}\right\} \text{cycloid.}$$

$(y - R\omega t)^2 + (z - R)^2 = R^2$ where $R = \frac{E_0}{\omega B_0}$.



- Work done by Lorentz force.

  charge q moves by $d\vec{l} = \vec{v}\,dt$ in $\vec{B}$.

  $$\vec{F}_q = q(\vec{v} \times \vec{B})$$

  $$\therefore\ dW = \vec{F}_q \cdot d\vec{l} = q\underline{(\vec{v}\times\vec{B})} \cdot \vec{v}\,dt = 0.$$

  mutually orthogonal

  Magnetic forces do no work.

$\boxed{\text{HW}}$ 5.1 & 5.3

- Lorentz Force on a Current-Carrying Wire.

  current $I = \dfrac{dq}{dt}$ and it's direction is $\begin{cases}\text{in the direction of (+) charge} \\ \text{in the opposite direction of (−) charge.}\end{cases}$

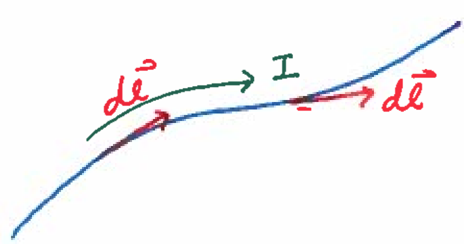
If a line charge λ travels along a wire at speed v, then $I = \dfrac{\lambda v \Delta t}{\Delta t} = \lambda v$. Therefore $\vec{I} = \lambda \vec{v}$

local velocity

Then the force on the wire in $\vec{B}$,

$$\boxed{\vec{F} = \int (\vec{v}\times\vec{B})\,dq = \int (\vec{v}\times\vec{B})\,\lambda\,dl = \lambda\int(\vec{v}\times\vec{B})\,dl = \int(\vec{I}\times\vec{B})\,dl}$$

(i) For a thin wire



$$d\vec{l} \parallel \vec{I}$$

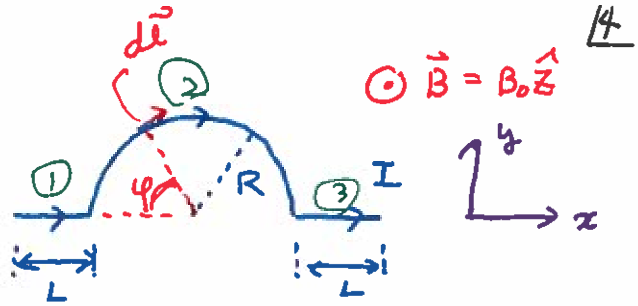
$$\therefore\ \boxed{\vec{F}_m = \int I(d\vec{l}\times\vec{B})}$$

Ex

Three sections of the wire

① & ③ straight

② half circle.



$$\vec{F}_1 = \vec{F}_2 = (\vec{I}\times\vec{B})L = I(\hat{x}\times\hat{z})BL = -IBL\,\hat{y}$$

$$\vec{F}_3 = I\int_{\wedge}(d\vec{l}\times\vec{B}) \underset{?}{=} I\int_0^{\pi} R\,d\varphi\,B_0\,(\hat{\varphi}\times\hat{z}) = -I\int_0^{\pi} RB_0\,d\varphi\,(-\hat{s})$$

Cylindrical coordinates (s, φ, z)

$$\hat{s} = -\cos\varphi\,\hat{x} + \sin\varphi\,\hat{y}$$

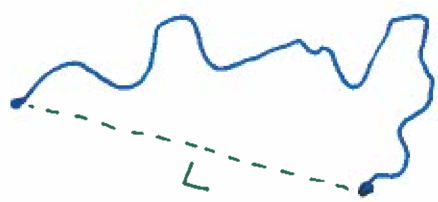
$$= -IRB_0\int_0^{\pi}(-\cos\varphi\,\hat{x} + \sin\varphi\,\hat{y})\,d\varphi$$

$$= -2IRB_0\,\hat{y}$$

You already know this by symm.

$$\therefore\ \vec{F}_m = -IB_0(2R+2L)\,\hat{y}$$

Simply the total straight length between the end points.

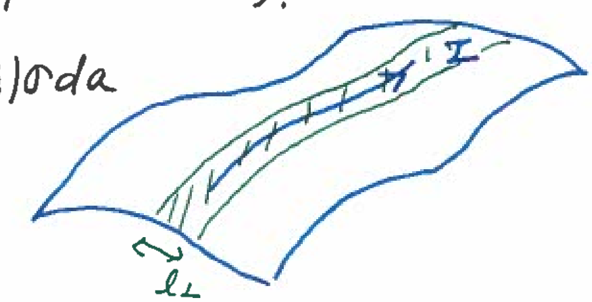


$$|\vec{F}_m| = IB_0L.$$

(ii) Surface current with $\vec{K}$ (surface current density).

$$\vec{K} = \sigma\vec{v} = \frac{d\vec{I}}{dl_\perp} \quad \text{(current per unit width).}$$

$$\therefore\ \vec{F}_m = \int(\vec{K}\times\vec{B})\underbrace{da}_{dl_\perp dl} = \int(\vec{v}\times\vec{B})\sigma\,da$$



(iii) Volume Current with $\vec{J}$ (vol. current density).

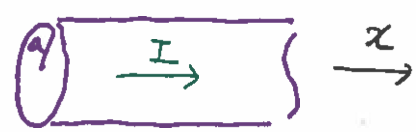
$$\vec{J} = \frac{d\vec{I}}{da_\perp} \quad \text{(current per unit area)}.$$

$$= \rho\vec{v}$$

$$\therefore \vec{F}_m = \int (\vec{J}\times\vec{B})\,d\tau = \int \rho(\vec{v}\times\vec{B})\,d\tau.$$

EX.

total current I.



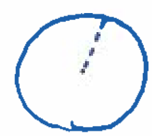
(a) Uniform current density

$$\vec{J} = \frac{I}{\pi a^2}\hat{x}$$

(b) If $J = ks$, then

$$I = \int_0^a s\,ds \int_0^{2\pi} d\varphi\,(ks).$$

$$= 2\pi k\left(\frac{a^3}{3}\right)$$

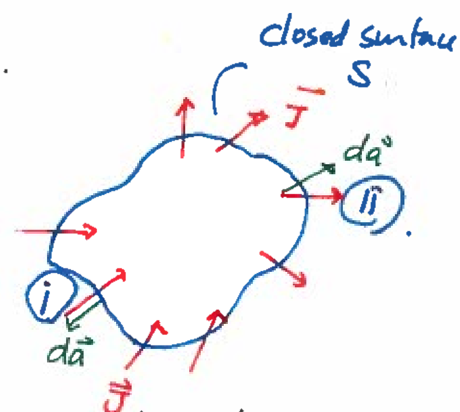


$$\therefore k = \frac{3I}{2\pi a^3}.$$

Continuity Eq. ("Charge Conservation")

$$I = \int_S \vec{J}\,da_\perp = \int_S \vec{J}\cdot d\vec{a} \quad (\approx \text{flux}).$$

For a closed surface,



$$\underbrace{\oint_S \vec{J}\cdot\vec{a}}_{} = \int_V (\vec{\nabla}\cdot\vec{J})\,d\tau$$

But this is related to charge accumulation in the volume.

(i) $\vec{J}\cdot d\vec{a} < 0$ will increase the charge in the volume.

(ii) $\vec{J}\cdot d\vec{a} > 0$ will decrease " " .

$$\therefore \int_V (\vec{\nabla}\cdot\vec{J})\,d\tau = -\frac{dQ_{en}}{dt} = -\frac{d}{dt}\int_V \rho\,d\tau = -\int \frac{\partial \rho}{\partial t}\,d\tau.$$

therefore $\boxed{\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0}$ Continuity Eq. $(\vec{J} = \rho\vec{v})$.

Any volume density ρ_K (K = mass, charge, $\cdots$).

Then the associated current density $\vec{J}_K = \rho_K \vec{v}$ local velocity.

If $\rho_K V = K$ (mass, charge, $\cdots$) is conserved,

$$\frac{\partial \rho_K}{\partial t} + \vec{\nabla}\cdot(\rho_K\vec{v}) = \frac{\partial \rho_K}{\partial t} + \vec{\nabla}\cdot\vec{J}_K = 0.$$

$\boxed{\text{HW}}$ 5.4, 5.5, 5.6.