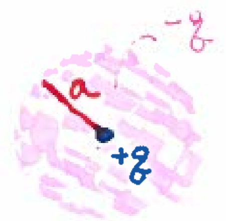
$\boxed{4.4}$ Ch.4 Electric Fields in Matter (Dielectric Materials).

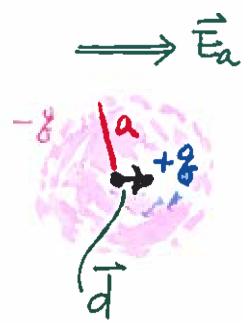
Insulators $\Leftrightarrow$ Dielectrics.

no free charges : localized electrons ~~by~~ around (+) nucleus. molecules

▸ Induced Dipoles.



apply $\vec{E} \neq 0$. $\Longrightarrow$



$\vec{E} = 0$

Symmetric distribution of electrons

$$\vec{p} = \int \vec{r}' \rho(\vec{r}') d\tau' = 0$$

- Consider $+q$ charge in this case. Two forces acting on $+q$ and now in equilibrium (balanced).
- Assuming the negative charge is uniformly distributed,

$$E_e(d) = \frac{1}{4\pi\varepsilon_0} \frac{qd}{a^3}.$$

- At $+q$, this field should cancel the applied field, $\vec{E}_a$
- Therefore

$$E_e(d) = E = \frac{1}{4\pi\varepsilon_0} \frac{qd}{a^3}$$

$$p = qd = 4\pi\varepsilon_0 a^3 E_a = \frac{4\pi}{3} a^3 \cdot (3\varepsilon_0) E_a = 3\varepsilon_0 \underline{V} E_a$$

$$= \alpha E_a$$

volume of an atom.

◎ Although this is based on a specific (simple) model, there is a general relation

$$\vec{p} = \underline{\underline{\alpha}} \vec{E}_a$$

applied field

polarizability

Induced dipole.

$$[\alpha] = \frac{[p]}{[E]} = \frac{[qd]}{[E]} = \frac{C \cdot m}{N/C} = C^2 m/N.$$

	H	He	Li	C	Ne	Na	K	Cs
$\boxed{\frac{\alpha}{4\pi\varepsilon_0} \times 10^{-30}\, m^3}$	0.667	0.205	24.3	~~5.60~~ 1.67	0.396	24.1	43.4	59.4

◎ For molecules, α is in general anisotropic.

$$\vec{p} = \alpha_\perp \vec{E}_\perp + \alpha_\parallel \vec{E}_\parallel . \neq \alpha \vec{E} \quad (\vec{p} \not\parallel \vec{E})$$

perpendicular or in parallel to the axis of molecule.

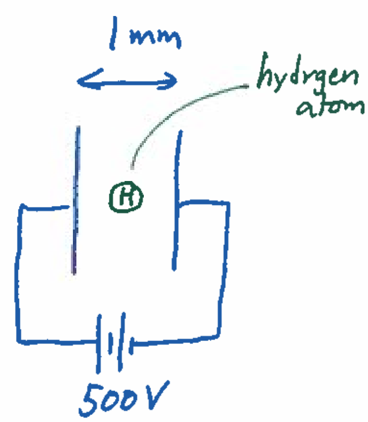
$$\uparrow \alpha_\perp$$
$$O = C = O \longrightarrow \alpha_\parallel$$

The most general linear relation between $\vec{E}$ & $\vec{P}$ is

$$\vec{P} = \overleftrightarrow{\alpha} \vec{E} = \underbrace{\begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}}_{\text{"Polarizability Tesor"}} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix}$$

EX A hydrogen atom in $\vec{E}$

$\alpha = 0.667$.



$$P = \alpha E = \alpha \left(\frac{V}{\theta}\right) = \alpha \frac{500}{10^{-3}} = \alpha\,(5\times10^{5})$$
$$= (0.677)(5\times10^{5})$$
$$= e \cdot d \sim \text{displacement}.$$

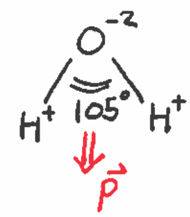
$$\therefore \quad d = \frac{(0.677)(5\times10^{5})}{1.6\times10^{-19}} \approx \underline{\underline{2\times10^{-16}}}\ (m).$$

tiny even relative to the size of ~~the~~ a hydrogen atom ($\sim 0.5\times10^{-10}$).

- Polar Molecules : $p \neq 0$ even $\vec{E}_a = 0$.

  - A dipole moment $\vec{p}$ experiences a torque in the presence of uniform $\vec{E}$.

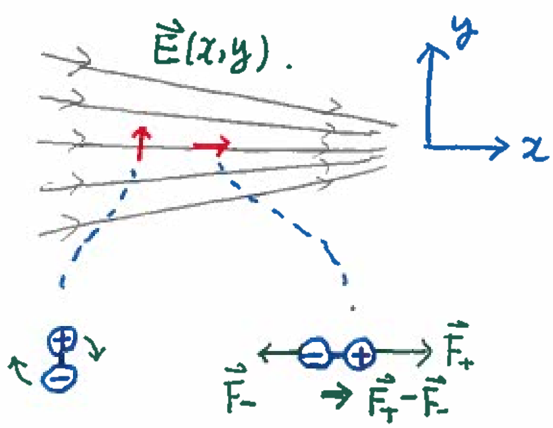
$$\boxed{\vec{N} = \vec{p} \times \vec{E}}$$



  - A dipole moment $\vec{p}$ experiences **No** net force in **uniform** $\vec{E}$.

  - A dipole moment $\vec{p}$ experiences **non-zero** net force in **non-uniform** $\vec{E}$.

$$\vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E}$$



$$\vec{E} = \vec{E}(x,y)$$

$$\vec{\nabla}\vec{E} = \vec{\nabla}E_x\hat{x} + \vec{\nabla}E_y\hat{y}$$

$$= \vec{\nabla}E_x(x,y)\hat{x} + \vec{\nabla}E_y(x,y)\hat{y}.$$

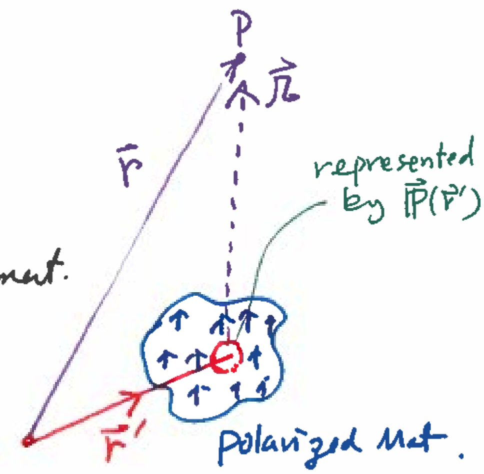
$\boxed{\text{HW}}$ 4.6, 4.7, 4.8.

Use image dipole to calculate the field.

* $\vec{p}$: dipole moment is a microscopic object. In a dielectric material there will be a macroscopic # of $\vec{p}$'s induced in response to $\vec{E}_{app}$. We can define a macroscopic quantity called "Polarization" to describe (quantify) this effect.

$\vec{\mathbb{P}}$: Polarization (total dipole moment per unit volume).

E. Potential & Field of a Polarized Object



$V(\vec{r})$ from a point dipole at $\vec{r}'$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p}\cdot\hat{\mathscr{r}}}{\mathscr{r}^2} \quad (\vec{\mathscr{r}} = \vec{r} - \vec{r}')$$

Extend this to continuously dist. dipole moment.

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\vec{\mathbb{P}}(\vec{r}')\cdot\hat{\mathscr{r}}}{\mathscr{r}^2}\, d\tau'$$

Using $\vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right) = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$; $\vec{\nabla}\left(\frac{1}{\mathscr{r}}\right) = -\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right)\cdot\vec{\mathbb{P}}(\vec{r}')\, d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0}\int_V \vec{\nabla}\cdot\left(\frac{\vec{\mathbb{P}}(\vec{r}')}{\mathscr{r}}\right) d\tau' - \frac{1}{4\pi\varepsilon_0}\int_V \frac{\vec{\nabla}'\cdot\vec{\mathbb{P}}(\vec{r}')}{\mathscr{r}}\, d\tau'$$

$\Leftarrow \boxed{\vec{\nabla}\cdot(f\vec{A}) = \vec{A}\cdot\vec{\nabla}f + f\vec{\nabla}\cdot\vec{A}}$

div. theorem

$$= \frac{1}{4\pi\varepsilon_0}\oint_S \frac{\vec{\mathbb{P}}(\vec{r}')}{\mathscr{r}}\cdot d\vec{a} + \frac{1}{4\pi\varepsilon_0}\int_V \frac{(-\vec{\nabla}'\cdot\vec{\mathbb{P}})}{\mathscr{r}}\, d\tau'$$

[$\sigma_b \equiv \vec{\mathbb{P}}\cdot\hat{n}$ and $\rho_b \equiv -\vec{\nabla}\cdot\vec{\mathbb{P}}(\vec{r})$]

<u>Bound</u> surf. charge density — <u>Bound</u> vol. charge density

$$= \frac{1}{4\pi\varepsilon_0}\oint \frac{\sigma_b(\vec{r}')}{\mathscr{r}}\, da' + \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho_b(\vec{r}')}{\mathscr{r}}\, d\tau'$$

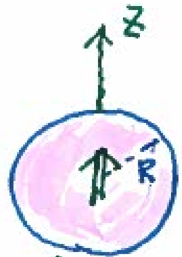
Polarized Object $\Rightarrow$ Surface charge + Vol. charge

$\Downarrow$

(i) when there is symm.
Use Gauss's Law
with $\sigma_b = \vec{\mathbb{P}}\cdot\hat{n}$ & $\rho_b = -\vec{\nabla}\cdot\vec{\mathbb{P}}$

or (ii) Laplace's Eq.

Ex



$\vec{P} = P_0\hat{z}$

$\Rightarrow$

$$\sigma_b = \vec{P}\cdot\hat{n} = \vec{P}\cdot\hat{r} = P_0\cos\theta$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P} = 0.$$

spherical shell with $\sigma_b = P_0\cos\theta$

This prob has been worked out in L12 p.6. for $\sigma(\theta) = k\cos\theta$.

$$\therefore \quad V(r,\theta) = \begin{cases} \dfrac{P_0}{3\varepsilon_0} r\cos\theta & \text{for } r \leq R \\ \dfrac{P}{3\varepsilon_0}\dfrac{R^3}{r^2}\cos\theta & \text{for } r \geq R. \end{cases}$$

$$\vec{E} = -\vec{\nabla}V$$

- For $r \leq R$, $\quad V(r,\theta) = \dfrac{P_0}{3\varepsilon_0} r\cos\theta = \dfrac{P_0}{3\varepsilon_0} z$.

$$\therefore \quad \vec{E} = -\frac{\partial V}{\partial z}\hat{z} = -\frac{P_0}{2\varepsilon_0}\hat{z} \quad \text{(uniform field).}$$

- For $r \geq R$, $\quad V(r,\theta) = \dfrac{\frac{4}{3}\pi R^3 P}{4\pi\varepsilon_0}\cdot\dfrac{\cos\theta}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{P\cos\theta}{r^2} = \underbrace{\dfrac{1}{4\pi\varepsilon_0}\dfrac{\vec{P}\cdot\hat{r}}{r^2}}_{\text{Single giant dipole moment}}$

total polarization.

$\boxed{\text{HW}}$ 4.10 (uniform σ_b and ρ_b).