L 3B

The Lagrangian for charge in the electric & magnetic field.

* The Lagrangian function $\mathcal{L}(q_i, \dot{q}_i, t)$ is not unique. For example: $\mathcal{L}' = \mathcal{L} + \text{const}$ or $\mathcal{L}' = \mathcal{L} + f$ where $\frac{\partial f}{\partial q_i} = \frac{d}{dt}\frac{\partial f}{\partial \dot{q}_i}$ produce the same EL equations.

* Any function $\mathcal{L}(q_i, \dot{q}_i, t)$ which gives correct EL equations of motion can be called Lagrangian.

* Newton's Law for charge e in $\vec{E}$ & $\vec{B}$

$$m\ddot{\vec{r}} = q(\vec{E} + \dot{\vec{r}} \times B)$$

scalar V, vector $\vec{A}$ potentials.

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\mathcal{L}(\vec{r}, \dot{\vec{r}}, t) = \frac{1}{2} m \dot{\vec{r}}^2 - q(V - \dot{\vec{r}} \cdot \vec{A})$$

equation for x: $\frac{\partial \mathcal{L}}{\partial x} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} \rightarrow$

$$\frac{\partial \mathcal{L}}{\partial x} = -q\left(\frac{\partial V}{\partial x} - \dot{x}\frac{\partial A_x}{\partial x} - \dot{y}\frac{\partial A_y}{\partial x} - \dot{z}\frac{\partial A_z}{\partial x}\right)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x} + qA_x = P_x \text{ — generalized moment}$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} = m\ddot{x} + q\left(\dot{x}\frac{\partial A_x}{\partial x} + \dot{y}\frac{\partial A_x}{\partial y} + \dot{z}\frac{\partial A_x}{\partial z} + \frac{\partial A_x}{\partial t}\right)$$

$$m\ddot{x} = -q\left(\frac{\partial V}{\partial x} + \frac{\partial A_x}{\partial t}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) - q\dot{z}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)$$

$$m\ddot{x} = qE_x + \dot{y}B_z - \dot{z}B_y$$

$$\vec{P} = m\vec{v} + q\vec{A}$$

QM: $P = -i\hbar\vec{\nabla} \rightarrow m\vec{v} = -i\hbar\vec{\nabla} - q\vec{A}$