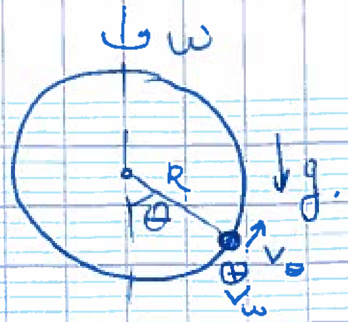
L34 Example 7.6



$v_\theta = R\dot{\theta}$

$v_\omega = R\omega\cos\theta = \rho\omega$

$\mathcal{L} = T - U = \frac{1}{2}mR^2(\dot{\theta}^2 + \omega^2\sin^2\theta) - mgR(1-\cos\theta)$

a) $\frac{\partial\mathcal{L}}{\partial\theta} = mR^2\omega^2\sin\theta\cos\theta - mgR\sin\theta = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\theta}} = mR^2\ddot{\theta}$

$\ddot{\theta} = (\omega^2\cos\theta - g/R)\sin\theta$

b) Equilibrium $\theta = 0$, $\theta = \pi$ $\theta = \pm\theta_0$

$\theta = \pi$ — always unstable.

$\theta = 0$ → case 1 $(\omega = 0)$ stable for $\omega < \sqrt{g/R}$

not stable case 2 $\omega > \sqrt{g/R}$ — bifurcation.

→ 2 stable positions at $\cos\theta_0 = g/\omega^2 R$

Example 7.7.

a) $\omega < \sqrt{g/R}$ $\ddot{\theta} = -(g/R - \omega^2)\theta$ $\theta \ll 1$

$\Omega = \sqrt{g/R - \omega^2}$ Ω is decreasing if ω increas.

b) for $\omega^2 = g/R$ the equilibrium @ $\theta = 0$ becomes unstable.

c) $\omega > \sqrt{g/R}$ $\ddot{\theta} = (g/R - \omega^2)\sin\theta$

use $\theta = \theta_0 + \varepsilon$ $\cos\theta_0 = g/R\omega^2$

$\cos(\theta_0 + \varepsilon) \approx \cos\theta_0 - \varepsilon\sin\theta_0$ $\sin(\theta_0 + \varepsilon) \approx \sin\theta_0 + \varepsilon\cos\theta_0$

$\ddot{\theta} = (\omega^2\cos(\theta_0 + \varepsilon) - g/R)\sin(\theta_0 + \varepsilon) =$

$= [\underbrace{\omega^2\cos\theta_0 - g/R}_{=0} - \varepsilon\omega^2\sin\theta_0] * (\sin\theta_0 + \varepsilon\cos\theta_0)$

$\ddot{\theta} = \ddot{\varepsilon} = -\varepsilon\omega^2\sin\theta^2 = -\Omega^2\varepsilon$

$\Omega = \sqrt{\omega^2 - (g/R\omega)^2}$

c) Ignorable coordinates