L31

Several unconstrained particles

2p: $\mathcal{L}(\vec{r}_1, \vec{r}_2, \dot{\vec{r}}_1, \dot{\vec{r}}_2) = \frac{1}{2}m_1\dot{\vec{r}}_1^2 + \frac{1}{2}m_2\dot{\vec{r}}_2^2 - U(\vec{r}_1, \vec{r}_2)$

6 EL equations for any set of 6 generalized coordinates $q_1 \dots q_6$

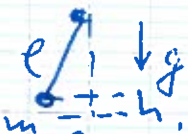
For example, we can use 3 coordinates for CM and 3 coordinates relative to CM.

$$R = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$$ – center of mass (CM)

* Constrained systems

if for N particles the # of degrees of freedom M, M<3N, then the system is constrained.

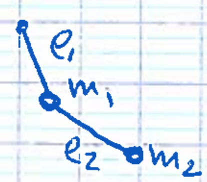
a) pendulum $x^2+y^2 = \ell^2 = \text{const.}$



out of 2 degrees of freedom, one is constrained as a free degree of freedom we can select:
1) x; $y = \sqrt{\ell^2 - x^2}$
2) y; $x = \sqrt{\ell^2 - y^2}$
3) φ; $x = \ell\sin\varphi$, $y = \ell\cos\varphi$

$$\mathcal{L}(\varphi) = T - U = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - mgh =$$
$$= \frac{1}{2}m\dot{\varphi}^2\ell^2 - mg\ell(1-\cos\varphi); \quad h = \ell(1-\cos\varphi)$$
$$\frac{\partial\mathcal{L}}{\partial\varphi} = -mg\ell\sin\varphi = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\varphi}} = m\ell^2\ddot{\varphi} \to$$
$$-mg\ell\varphi = m\ell^2\ddot{\varphi} \to \ddot{\varphi} = -\frac{g}{\ell}\varphi$$

b)

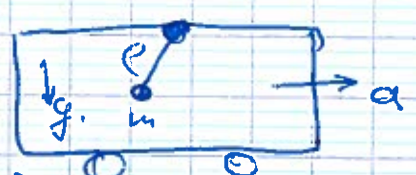


double pendulum → 2 DOF

DOF = degrees of freedom

$$\vec{r}_1 = (\ell_1\sin\varphi_1, \ell_1\cos\varphi_1) = \vec{r}_1(\varphi_1)$$
$$\vec{r}_2 = (\ell_1\sin\varphi_1 + \ell_2\sin\varphi_2, \ \ell_1\cos\varphi_1 + \ell_2\cos\varphi_2) = \vec{r}_2(\varphi_1, \varphi_2)$$

c) accelerating railcar



$$\vec{r} = (x, y) = \left(\ell\sin\varphi + \frac{at^2}{2}, \ell\cos\varphi\right)$$
$$\vec{V} = (\ell\dot{\varphi}\cos\varphi + at, -\ell\dot{\varphi}\sin\varphi)$$
$$\frac{1}{2}mV^2 - U = \frac{m}{2}\ell^2\dot{\varphi}^2 + \frac{m}{2}a^2t^2 + m\ell at\dot{\varphi}\cos\varphi - mg\ell(1-\cos\varphi)$$
$$\frac{\partial\mathcal{L}}{\partial\varphi} = -m\ell\dot{\varphi}at\sin\varphi - mg\ell\sin\varphi =$$
$$= \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\varphi}} = m\ell^2\ddot{\varphi} + m\ell a\cos\varphi - m\ell\dot{\varphi}at\sin\varphi$$