L18. Central forces $F(\vec{r}) = f(\vec{r})\,\hat{r}$

Coulomb force $\frac{kqQ}{r^2}\hat{r}$ $\quad k = \frac{1}{4\pi\varepsilon_0}$

a) conservative

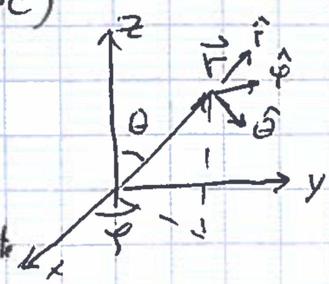
b) Spherically symmetric. $f(\vec{r}) = f(r)$

* Spherical Polar Coordinates. (SPC)

$z = r\cos\theta$

$x = r\sin\theta\cos\varphi \quad y = r\sin\theta\sin\varphi$

θ gives latitude, φ is the longitude — Earth polar coordinate



$f(r)$ spherically symetric $\equiv$ does not depend on θ and φ. $\frac{\partial f}{\partial\theta} = \frac{\partial f}{\partial\varphi} = 0$

unit vectors: $\hat{r}, \hat{\theta}, \hat{\varphi}$ - orthogonal

$$\vec{a} = a_r\hat{r} + a_\theta\hat{\theta} + a_\varphi\hat{\varphi}$$

* Gradient in SPC

$$\vec{\nabla} f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z} \rightarrow \vec{\nabla} f\cdot d\vec{r} = df \quad (a)$$

$$d\vec{r} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\varphi\,\hat{\varphi} \quad (b)$$

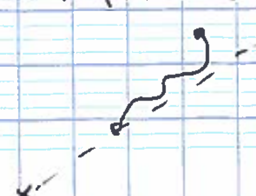
combine (a) and (b)

$$df = \underbrace{(\vec{\nabla} f)_r}_{\frac{\partial f}{\partial r}} dr + \underbrace{(\vec{\nabla} f)_\theta r}_{\frac{\partial f}{\partial\theta}} d\theta + \underbrace{(\vec{\nabla} f)_\varphi r\sin\theta}_{\frac{\partial f}{\partial\varphi}} d\varphi$$

$$\vec{\nabla} f = \hat{r}\frac{\partial f}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial f}{\partial\theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial f}{\partial\varphi}$$

a) a central force is conservative if it is spherically symmetric: $F(\vec{r}) = -\vec{\nabla} U = -\hat{r}\frac{\partial U}{\partial r} = -\hat{r} f(r)$

b) if central force is spherically symmetric $\Rightarrow$ conservative



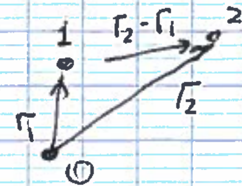
- any path can be presented as shifts along $\hat{r}$ and $\perp$ to $\hat{r}$

Problem 4.43 & 4.44

L18 Interaction of two particles.

consider central forces and $\vec{F}_{12} = -\vec{F}_{21}$

* Binary stars:



$F_{12} = \frac{G m_1 m_2}{(\vec{r}_1 - \vec{r}_2)^2} \hat{r}$

$\hat{r}$ - unit vector along $\vec{r}_1 - \vec{r}_2$

The result is the same if we move point O → lets move O into particle 2 location. The force on 1 is conservative: $\vec{\nabla}_1 \times \vec{F}_{12} = 0$

$\vec{\nabla}_1 = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial y_1}, \frac{\partial}{\partial z_1}\right)$ also $\vec{F}_{12} = -\vec{\nabla}_1 U(\vec{r}_1)$

or $\vec{F}_{12} = -\vec{\nabla}_1 U(\vec{r}_1 - \vec{r}_2)$ for arbitrary O

$\vec{F}_{21} = -\vec{\nabla}_2 U(\vec{r}_1 - \vec{r}_2)$

because $\vec{\nabla}_1 U(r_1 - r_2) = -\vec{\nabla}_2 U(r_1 - r_2)$

derive. forces as a gradient of a potential energy U at locatio 1 and 2.

* Conservation of total mechanical energy.

$dT_1 = d\vec{r}_1 \cdot \vec{F}_{12}$

$dT_2 = d\vec{r}_2 \cdot \vec{F}_{21}$

$dT = dT_1 + dT_2 =$

$= d(\vec{r}_1 - \vec{r}_2)\, \vec{F}_{12} =$

$dT = d(\vec{r}_1 - \vec{r}_2)\left[-\vec{\nabla}_1 U(r_1 - r_2)\right] = -d\vec{r} \cdot \vec{\nabla} U(r) = -dU$

$dT + dU = d(T + U) = 0$

$E = T + U = T_1 + T_2 + U = \text{const.}$