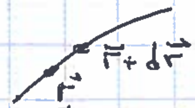
2.14 Gradient of $U(\vec r)$



$\Delta W(\vec r \to \vec r + d\vec r) = \vec F \cdot d\vec r = -[U(\vec r + d\vec r) - U(\vec r)]$

$= F_x dx + F_y dy + F_z dz$

$= -[U(x+dx, y+dy, z+dz) - U(x,y,z)] =$

$= -\left[\frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy + \frac{\partial U}{\partial z}dz\right]$ $\quad \boxed{\frac{\partial U}{\partial x} \text{ - partial derivative}}$

$x, y, z \to$ independent variables

$\Delta W = -dU =$
$= -\vec\nabla U\, d\vec r$
$dU = \vec\nabla U\, d\vec r$
$\frac{dU}{d\vec r} = \vec\nabla U$

$\frac{\partial U(x,y,z)}{\partial x} = \frac{\partial U(x)}{\partial x}$ $(y = \text{const}, z = \text{const})$

$\vec F = -\hat x \frac{\partial U}{\partial x} - \hat y \frac{\partial U}{\partial y} - \hat z \frac{\partial U}{\partial z} = -\vec\nabla U$

Operator gradient $\vec\nabla = \hat x \frac{\partial}{\partial x} + \hat y \frac{\partial}{\partial y} + \hat z \frac{\partial}{\partial z}$

– example: $U(r) = \frac{GmM}{r}$ $\quad r = \sqrt{x^2+y^2+z^2}$

$U(r)$ – gravitational potential energy

$\frac{\partial U}{\partial x} = -\frac{GmM}{r^2}\frac{\partial r}{\partial x} = -\frac{GmM}{r^3}x$

$\vec F = \frac{GmM}{r^3}(\vec x + \vec y + \vec z) = \frac{GmM}{r^3}\vec r$

$\boxed{\begin{array}{l} U(r) = \varphi(r) m \\ \varphi(r) \text{ – potential} \end{array}}$

same for Coulomb Force

– Curl operator. (Stock's theorem) problem 4.25

line integral $\int_1^2 \vec F d\vec r$ does not depend on the pass between 1 & 2

this is equivalent to $\vec\nabla \times \vec F = 0 \to$ curl of F

Coulomb field:

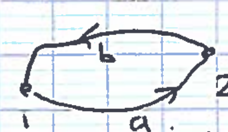
$\varphi_c(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

$\vec F_q = -q\vec\nabla\varphi$

$\vec\nabla$	$\frac{\partial}{\partial x}$	$\frac{\partial}{\partial y}$	$\frac{\partial}{\partial z}$
$\vec F$	F_x	F_y	F_z
$\vec\nabla\times\vec F$	$\frac{\partial}{\partial y}F_z - \frac{\partial}{\partial z}F_y$	$\frac{\partial}{\partial z}F_x - \frac{\partial}{\partial x}F_z$	$\frac{\partial}{\partial x}F_y - \frac{\partial}{\partial y}F_x$

– Problem 4.25

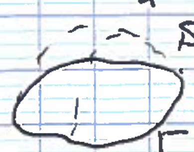
$\vec\nabla \times \vec F_q = 0$



$\Gamma = a + b$ closed path

$\oint_\Gamma \vec F d\vec r = \int_a \vec F d\vec r + \int_b \vec F d\vec r = 0$ for conservative force

We can reverse the problem: given $\vec\nabla \times \vec F = 0$ we can find $U(\vec r)$



S' - arbitrary surface associated with Γ

Stock's Theorem $\oint_\Gamma \vec F d\vec r = \iint_S \vec\nabla \times \vec F d\vec S$

$d\vec S = |dS|\hat n$ – element of the surface S with area $|dS|$ and $\hat n \perp$ to the surface.

$\iint_S \vec\nabla \times \vec F d\vec S = 0$ for any surface S $\to \vec\nabla \times \vec F = 0$