L12 Angular Momentum of Several Particles.

* $\vec{L} = \sum_{\alpha=1}^{N} \ell_\alpha = \sum_\alpha \vec{r}_\alpha \times \vec{p}_\alpha \qquad \dot{\vec{L}} = \sum_\alpha \vec{r}_\alpha \times \vec{F}_\alpha$

separate "internal" and "external" forces:

$\vec{F}_\alpha = \sum_{\alpha \neq \beta} \vec{F}_{\alpha\beta} + \vec{F}^{\alpha}_{ext}$

$\dot{\vec{L}} = \sum_\alpha \sum_{\beta \neq \alpha} \vec{r}_\alpha \times \vec{F}_{\alpha\beta} + \sum_\alpha \vec{r}_\alpha \times F^{\alpha}_{ext}$

for each $\alpha\beta$ pair write: $\vec{r}_\alpha \times \vec{F}_{\alpha\beta} + \vec{r}_\beta \times \vec{F}_{\beta\alpha}$

using the third law: $\vec{F}_{\alpha\beta} = -\vec{F}_{\beta\alpha}$

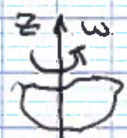
so $(\vec{r}_\alpha - \vec{r}_\beta) \times \vec{F}_{\alpha\beta} = 0$ for the central force $\vec{F}_{\alpha\beta} \parallel$ to $(\vec{r}_\alpha - \vec{r}_\beta)$

$\dot{\vec{L}} = \sum_\alpha \vec{r}_\alpha \times F^{\alpha}_{ext} = \sum \vec{\Gamma}^{\alpha}_{ext} = \vec{\Gamma}_{ext}$

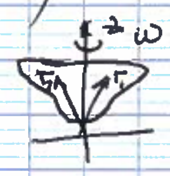
— Conservation of angular momentum: $\dot{\vec{L}} = 0$ if $\vec{\Gamma}_{ext} = 0$

* Moment of Inertia

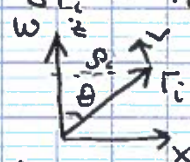
consider a motion (rotation) of a rigid body — individual particles do not move wrt each other.



a) symmetric wrt the rotation axis



$d\vec{\ell}_i = \vec{r}_i \times [\vec{\omega} \times \vec{r}_i]\, dm_i =$



$d\ell^i_z = \omega r_i^2 \sin^2\theta\, dm =$

$= \omega \rho_i^2\, dm$

$d\ell^i_x = -\omega r_i^2 \sin\theta \cos\theta\, dm$

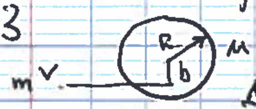
When integrate over all masses

$L_z = \sum dm_i \rho_i^2 \omega = I\omega \qquad L_x = 0$

Moment of Inertia $I = \sum m_i \rho_i^2$

ρ_i — distanse to the rotation axis.

Example 3.3



mass per unit area $= M/\pi R^2 = dm/dA$.

$\Delta m = 2\pi r\, dr \cdot \frac{M}{\pi R^2} \qquad \Delta I = \Delta m\, r^2$

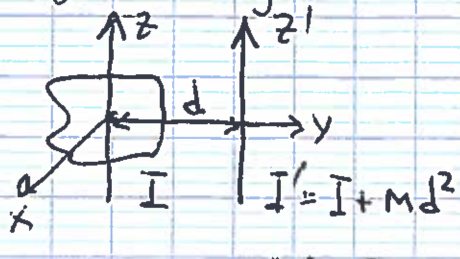
$I = \int_0^R \frac{M}{R^2} 2r^3 dr = \frac{M}{R^2} R^4/2 = M R^2/2$

$L_z = mvb = (m + \frac{M}{2}) R^2 \omega \qquad V = \frac{m}{m + M/2} \frac{V b}{R^2}$

L12

Parallel axis (Huygens-Steiner) theorem

Defines moment of inertia of a rigid body about any parallel axis.

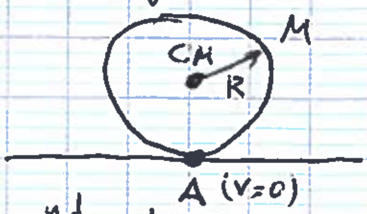


$$I = \sum dm_i \rho_i^2 = \sum dm_i (x_i^2 + y_i^2)$$

$$I' = \sum dm_i \left[x_i^2 + (y_i + d)^2\right] =$$

$$= \underbrace{\sum dm_i (x_i^2 + y_i^2)}_{I} + \underbrace{\sum dm_i\, d^2}_{Md^2} + \underbrace{\sum dm_i\, y d}_{y_{cm} = 0}$$

Rolling wheel (not slipping)

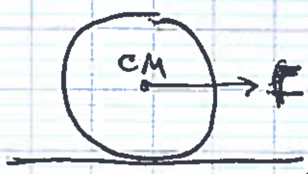


$I_{CM} = MR^2$ (about CM)

$I_A = MR^2 + MR^2 = 2MR^2$ (about A)

2^{nd} N's law for rotation is $\dot{\vec{L}}_{cm} = \vec{\Gamma}_{cm}$ even if CM is not an inertial ref. frame.

$$\dot{L}_{cm} = I_{cm}\dot{\omega} = \Gamma_{cm}$$



apply force F to CM

Newton's laws:

(1) $m\dot{v} = F - f$ f - friction force

(2) $MR^2\dot{\omega} = fR$

F torque about CM = 0

$f = m\dot{v} = F - f$ $f = F/2$