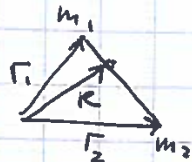


L11 the center of Mass.

$$\vec{R} = \frac{1}{\sum m_i} \sum m_i \vec{r}_i \rightarrow (X, Y, Z)$$

2 particles:
$$\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$



- \vec{R} ends on the line connecting m_1 & m_2

$$\vec{r}_1 - \vec{R} = \frac{m_2}{m_1 + m_2} (\vec{r}_1 - \vec{r}_2) \quad \vec{r}_2 - \vec{R} = -\frac{m_1}{m_1 + m_2} (\vec{r}_1 - \vec{r}_2)$$

-
$$\frac{d}{dt} (\sum m_i \vec{R} = M \vec{R} = \sum m_i \vec{r}_i) \rightarrow M \dot{\vec{R}} = \sum m_i \dot{\vec{r}}_i = \vec{P}$$

 \vec{P} is total momentum. So

$$\vec{F}_{ext} = M \ddot{\vec{R}}$$

- Lab and CM reference frames (Example with elastic collic)

- Calculation of CM.

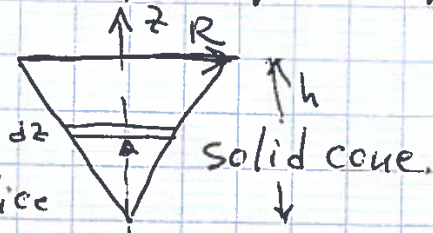
$$R = \frac{1}{M} \sum dm_i \vec{r}_i \Rightarrow \frac{1}{M} \int \vec{r} dm = \frac{1}{M} \int \rho \vec{r} dV$$

Use different coordinate systems (Cartesian, cyl. Spherical) depending on the symmetry of the object

- Example 3.2

a) CM should be on z axis

b) mass of the circular slice at z with dz thickness



$$dm = \rho \pi r^2(z) dz = \rho \pi R^2 \frac{z^2}{h^2} dz$$

c) CM:
$$z = \frac{1}{M} \sum dm_i z_i = \frac{1}{M} \int_0^h z dm = \frac{1}{M} \frac{\rho \pi R^2}{h^2} \int_0^h z^3 dz = \frac{\rho \pi R^2 h^4}{4 M h^2}$$

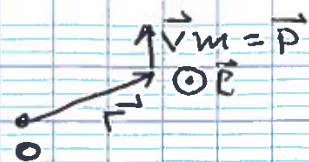
d) What is M?
$$M = \sum dm_i = \int_0^h \frac{\rho \pi R^2}{h^2} z^2 dz = \frac{\rho \pi R^2}{3 h^2} h^3$$

e)
$$z = \frac{3}{4} h$$

L11

Angular momentum

$$\vec{\ell} = \vec{r} \times \vec{p}$$



$$\dot{\vec{\ell}} = (\dot{\vec{r}} \times \vec{p}) + \vec{r} \times \dot{\vec{p}} = \dot{\vec{r}} \times \vec{p} + \vec{r} \times \vec{F} = \vec{r} \times \vec{F} = \vec{\tau} \text{ - torque}$$

$$\dot{\vec{\ell}} = \vec{\tau} \text{ - rotational version of Newton's 2nd law}$$

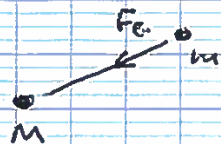
- Central force (like gravity)

can select origin when $\vec{\tau} = 0$

$$\ell = \text{const.}$$

conservation of angular momentum

$\vec{r}(t) \times \vec{p}(t)$ defines a plane



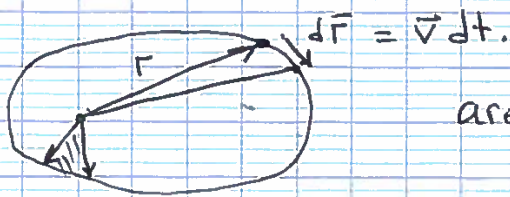
- Kepler's laws - summaries of observed planetary motion (empirical)

1st - planet orbits are ellipses.

2nd - a line segment sweeps equal areas in equal times. ($dA/dt = \text{const}$)

3rd - $T^2 \propto a^3$ T - period a - semi-major

- Kepler's Law can be derived from Newton's Laws
postpone 1st & 3rd law till later.



area of a triangle

$$a \sin \theta = h$$

$$\frac{1}{2} |a \times b| = \frac{a b \sin \theta}{2} = A$$

$$\frac{b}{2} = A$$

$$\Delta A = \frac{1}{2} [\vec{r} \times \vec{v} dt] = \frac{1}{2m} [\vec{r} \times \vec{p}] dt$$

$$\frac{dA}{dt} = \frac{1}{2m} [\vec{r} \times \vec{p}] = \vec{\ell}$$