L9 Complex exponentials

* $e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} \dots$

$$\frac{\partial e^z}{\partial z} = 0 + 1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} \dots = e^z$$

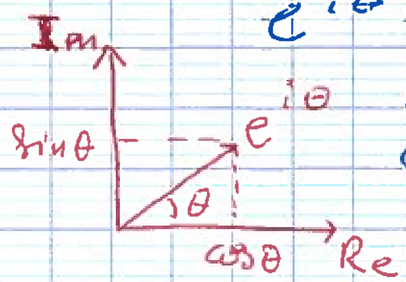
now Let $z = i\theta$

$$e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2} \dots = 1 + i\theta - \frac{\theta^2}{2!} + i\frac{\theta^3}{3!} - i\frac{\theta^4}{4!} \dots$$

$$= \left[1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!}\right] + i\left[\theta - \frac{\theta^3}{3!} - \frac{\theta^5}{5!}\right]$$

$\cos\theta = 1 - 0 - \frac{\theta^2}{2!} + 0 + \frac{\theta^4}{4!}$ $\quad \sin\theta$

$$e^{i\theta} = \cos\theta + i\sin\theta \quad \text{Euler's form}$$

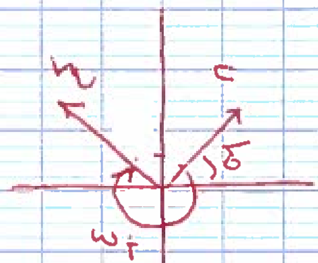


any complex number $z = a + ib$ can be presented as $|z|e^{i\theta} =$

$= |z|(\cos\theta + i\sin\theta)$

$a = |z|\cos\theta$

$b = |z|\sin\theta$ $\quad |z| = \sqrt{a^2 + b^2}$



$$\eta = ce^{-i\omega t} = |c|e^{i\delta}e^{-i\omega t}$$
$$= |c|e^{-i(\omega t - \delta)}$$

* Solution for charge in magnetic field

angular velocity $\omega = \frac{qB}{m}$

complex coordinate $\zeta = x + iy$ (t)

$\eta = \dot{\zeta} = \dot{x} + i\dot{y} = V_x + iV_y$

$\zeta = \int \eta(t)\,dt = \int Ae^{-i\omega t}dt = -\frac{A}{i\omega}e^{-i\omega t} + const$

Select coordinate system so const=0

$\zeta = \frac{iA}{\omega}e^{-i\omega t} = \frac{A}{\omega}e^{-i\omega t + \pi/2}$

* Larmor radius $r = \frac{v}{\omega} = \frac{mv}{qB} = \frac{p}{qB}$ —

used to measure particle's momentum in High Energy experiments at CERN.