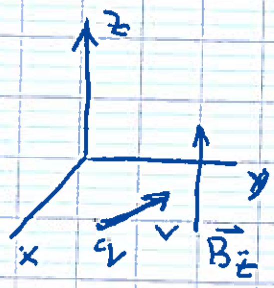
L8

Motion of charge in a Uniform mag field.



$$\vec{F} = q\,\vec{v}\times\vec{B} = m\dot{\vec{v}}$$

to simplify (no loss of generality)

select $B = (0,0,B)$, $V = (V_x, V_y, 0)$

$$\vec{V}\times\vec{B} = \det\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ V_x & V_y & 0 \\ 0 & 0 & B \end{vmatrix} = (V_y B, -V_x B, 0)$$

$$m\dot{\vec{V}} = q\,\vec{V}\times\vec{B} \longrightarrow \begin{aligned} m\dot{V}_x &= qBV_y \rightarrow \dot{V}_x = \omega V_y \\ m\dot{V}_y &= -qBV_x \rightarrow \dot{V}_y = -\omega V_x \\ m\dot{V}_z &= 0 \end{aligned}$$

$$\omega = \frac{qB}{m}$$

cyclotron frequency

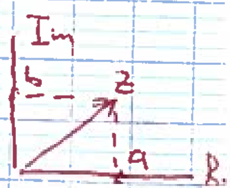
oscillator

Solution 1: $\frac{\partial}{\partial t}(\dot{V}_x = \omega V_y) = \ddot{V}_x = \omega\dot{V}_y = -\omega^2 V_x$

$\frac{\partial}{\partial t}(\dot{V}_y = -\omega V_x) = \ddot{V}_y = -\omega\dot{V}_x = -\omega^2 V_y$

The solutions are $A\sin\omega t$ & $B\cos\omega t$.

Solution 2: $\eta = \underset{Re}{V_x} + i\underset{Im}{V_y}$ $\quad i = \sqrt{-1}$



η is the complex velocity

any complex number $z = \underset{Re}{a} + i\underset{Im}{b}$ can be described by a vector

$$\dot{\eta} = \dot{V}_x + i\dot{V}_y = \omega V_y - i\omega V_x$$
$$= -i\omega(V_x + iV_y) = -i\omega\eta$$

two real equations are equivalent to one compx

$$\boxed{\dot{\eta} = -i\omega\eta} \equiv \begin{aligned} \dot{V}_x &= \omega V_y \\ \dot{V}_y &= -\omega V_x \end{aligned}$$

$i\omega = \text{const}$ $\quad \eta = ce^{-i\omega t}$ = complex solution

L9: show that $A\sin\omega t$ & $B\cos\omega t$ is equivalent to $c\,e^{i\omega t}$, c - complex number.