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Air resistance (drag)

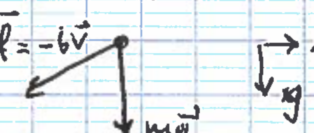
- drag $F(v)$ depends on the object velocity V
  we will consider that $\vec{F}(v) = -f(v)\,\hat{v}$
  Examples when it is not true.
- airplane, spinning ball, sail boat, ...
- consider case when $v \ll S$. (speed of sound)
  $\boxed{f(v) = bv + cv^2}$ – Taylor expansion over v/s

  $f_{lin} = bV$ – linear (viscous) drag $= \beta D V$
  $f_{quad} = cV^2$ – quadratic drag $= \gamma D^2 V^2$
  For spherical projectile: $\beta = 1.6\,10^{-4}\ Ns/m^2$
  in STP air $\gamma = 0.25\ Ns^2/m^4$

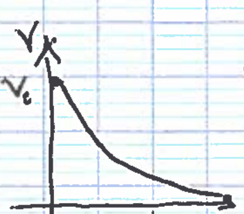
- Reynolds number: $\boxed{R = Dv\,\frac{\rho}{\eta}}$ ← density, ← viscosity
  R is a rough approximation for $\frac{f_{quad}}{f_{lin}} = \frac{\gamma}{\beta} DV$

- Example 2.1
  Baseball ($D = 7\,cm$, $V \sim 5\,m/s$) $\frac{f_q}{f_e} \sim 570$
  Millikan oil drop ($D = 1.5\,\mu m$, $V \sim 10^{-4}\,m/s$) $\frac{f_q}{f_e} \sim 10^{-7}$
  f_q is dominant when R is large
  f_e is dominant when R is small

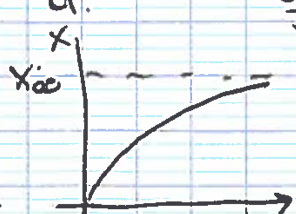
- Linear air resistance



  $\boxed{m\dot{\vec{V}} = m\vec{g} - b\vec{V}}$
  $m\dot{V}_x = -bV_x$ } independent equations
  $m\dot{V}_y = mg - bV_y$ } for x & y components.
- solution for x: $\dot{V}_x = -\frac{b}{m}V_x = -kV_x$
  or. $\frac{dV_x}{V_x} = -k\,dt \rightarrow \int \frac{dV_x}{V_x} = \int -k\,dt$
  $V_x = V_0 e^{-kt} \leftarrow \ln\frac{V_x}{V_0} = -kt$



$V_0 e^{-kt}$



$V_0/k(1 - e^{-kt})$

L6 Solution for y: case a) $\uparrow V_y = V_0$ $\downarrow g$

$\dot{V}_y = g - \frac{b}{m} V_y$ case b) $\downarrow V_y = V_0$

regardless how the motion starts (initial cond.) the projectile will be falling down and reach its terminal velocity $V_t = \frac{mg}{b}$.

* V_t for small drops

$b = \beta D$ $m = \rho \pi D^3/6$ $\boxed{V_t = \frac{\rho \pi D^2}{6\beta} g}$

$\beta = 1.6 \cdot 10^{-4}$ $V_{ter} = \frac{840 \cdot \pi (1.5 \cdot 10^{-6})^2}{6(1.6 \cdot 10^{-4})} \cdot 9.8 \approx 6 \cdot 10^{-5}$ m/s.

case b) $\dot{V}_y = \frac{b}{m}(V_t - V_y) = -\frac{b}{m}(V_y - V_t)$

$u = V_y - V_t$ $\frac{du}{dt} = \frac{dV_y}{dt} = \dot{V}_y = \dot{u}$ $\rightarrow$ $\dot{u} = -\frac{b}{m} u$

same equation as for X case: $u = A e^{-kt}$

$V_y - V_t = (V_{y0} - V_t) e^{-kt}$ defined A from initial conditions

$V_y(t) = V_{y0} e^{-kt} + V_t (1 - e^{-kt})$

$V_y(\infty) = V_t.$

* characteristic time $V_t = g/k = g\tau$

$V_{y0} = 0$ $\longrightarrow$ $V_y(t) = V_t (1 - e^{-t/\tau})$

in $t = \tau$ a falling object reaches 63% of V_t

or τ is time when object reaches V_t if accelerated with const $a = g$.

case a) Case a) $y\uparrow$ $\uparrow V_{y0}$ reverse y axis $\rightarrow V_t \rightarrow -V_t$.

$V_y(t) = V_{y0} e^{-t/\tau} - V_t (1 - e^{-t/\tau})$

$V_y(0) = V_{y0}$

$V_y(\infty) = -V_t$

$y(t) = \int_0^t V_y \, dt = -V_t t + (V_{y0} + V_t)\tau \times (1 - e^{-t/\tau})$

projectile reaches max elevation

$y_{max} = y(t_m)$ when $V_y(t_m) = 0$