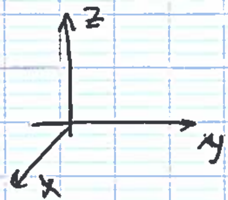
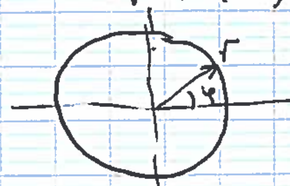
L5

Cartesian



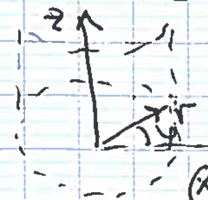
(x, y, z)

Polar (2D)



$(x,y) \to (r, \varphi)$

Cylindrical
Polar + z

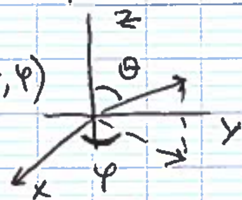


$z = z$
$x = r\cos\varphi$
$y = r\sin\varphi$

$(x,y,z) \to (r, \varphi, z)$

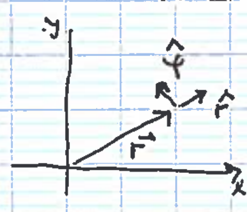
Spherical

$(x,y,z) \to (r,\theta,\varphi)$



$z = r\cos\theta$
$x = r\sin\theta\cos\varphi$
$y = r\sin\theta\sin\varphi$

– 2D Polar coordinas



$\vec{F} = F_r\,\hat{r} + F_\varphi\,\hat{\varphi}$

Line element: $d\vec{s} = dr\,\hat{r} + r\,d\varphi\,\hat{\varphi}$

$\Delta\vec{s} = \vec{r}(t+\Delta t) - \vec{r}(t)$

Velocity $\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}} = \left(\frac{\Delta s}{\Delta t}\right)_{\Delta t \to 0} = \dot{r}\,\hat{r} + r\dot{\varphi}\,\hat{\varphi}$

acceleration $\vec{a} = \ddot{\vec{r}} = \frac{d\dot{\vec{r}}}{dt} = (\ddot{r} - r\dot{\varphi}^2)\,\hat{r} + (r\ddot{\varphi} + 2\dot{r}\dot{\varphi})\,\hat{\varphi}$

used: $\boxed{\frac{d\hat{r}}{dt} = \dot{\varphi}\,\hat{\varphi} \qquad \frac{d\hat{\varphi}}{dt} = -\dot{\varphi}\,\hat{r}}$ (see textbook) Fig 1.13

– Newton's 2nd Law in polar coordinate

$\vec{F} = m\vec{a} \quad \to \quad F_r = m(\ddot{r} - r\dot{\varphi}^2)$ centripetal

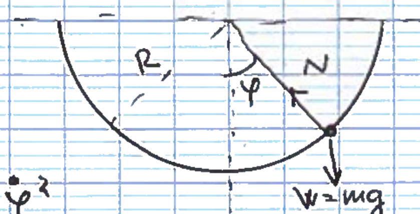
$F_\varphi = m(r\ddot{\varphi} + 2\dot{r}\dot{\varphi})$ Coriolis

$F_z = m\ddot{z}$ (cylindrical)

$r = \text{const}$ $\quad F_r = -mr\dot{\varphi}^2 = -r\omega^2 m$

$z = \text{const.}$ $\quad F_\varphi = mr\dot{\omega}$

L5 Example 1.2 An Oscillating Skateboard.



$r = R$

$F_r = -mR\dot{\varphi}^2$ $\quad$ $F_r = mg\cos\varphi - N$

$F_\varphi = mR\ddot{\varphi}$ $\quad = \quad$ $F_\varphi = -mg\sin\varphi$

$$-mg\sin\varphi = mR\ddot{\varphi}$$

$$\ddot{\varphi} = -g/R \sin\varphi$$

$$\ddot{\varphi} \approx -g/R\, \varphi \qquad \varphi \ll 1.$$

define $\omega = \sqrt{g/R}$ $\quad \ddot{\varphi} = -\omega^2\varphi$

solutions: $\varphi_1(t) = A\sin\omega t$

$\varphi_2(t) = B\cos\omega t$

$\varphi(t) = \varphi_1(t) + \varphi_2(t)$ – general solution.

- consider initial conditions.
- period of oscillations: $\tau = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$