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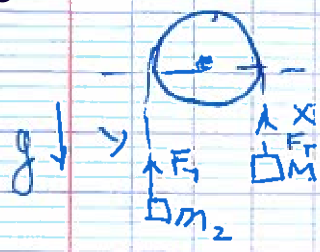
Example 7.3 Atwood's Machine



$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \dot{y}^2$$

$$x + y = \text{const} \rightarrow \dot{x} = -\dot{y}$$

$$T = \frac{1}{2}(m_1 + m_2)\, \dot{x}^2$$

$$U = -m_1 g x - m_2 g y = -(m_1 - m_2) g x + C$$

set $C = 0$ — not important.

$$\mathcal{L} = T - U = \frac{1}{2}(m_1 + m_2)\dot{x}^2 + (m_1 - m_2) g x$$

EL eq: $(m_1 - m_2) g = (m_1 + m_2)\ddot{x}$;

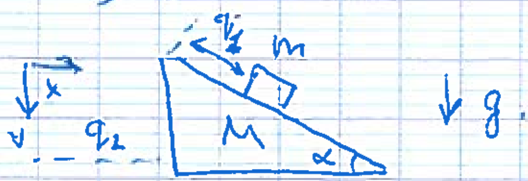
$$\ddot{x} = \frac{(m_1 - m_2)}{m_1 + m_2} g$$ — can be used to measure g.

Just Newton's Laws:

a) $m_1 \ddot{x} = m_1 g - F_T$ $\rightarrow$ a+b $= (m_1+m_2)\ddot{x} = (m_1 - m_2) g$.

b) $+\; m_2 \ddot{x} = +(F_T - m_2 g)$

Example 7.5



$$T_M = \frac{1}{2} M \dot{q}_2^2 \qquad \vec{v}_m = (\dot{q}_1 \cos\alpha + \dot{q}_2,\ \dot{q}_1 \sin\alpha)$$

$$T = T_M + T_m \ ; \quad T_m = \frac{1}{2} m v_m^2$$

$$T = \frac{1}{2}(M+m)\dot{q}_2^2 + \frac{1}{2} m\left(\dot{q}_1^2 + 2\dot{q}_1\dot{q}_2 \cos\alpha\right)$$

$$U = U_M + U_m = 0 + (-m g q_1 \sin\alpha)$$

q_2: $\mathcal{L} = T - U$ does not depend on q_2 so $\frac{\partial \mathcal{L}}{\partial q_2} = 0$

and $\frac{\partial \mathcal{L}}{\partial \dot{q}_2} = \text{const} = (M+m)\dot{q}_2 + m\dot{q}_1 \cos\alpha = P_x$

conservation of x component of momentum.

q_1: $$\frac{\partial \mathcal{L}}{\partial q_1} = m g \sin\alpha = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_1} = \frac{d}{dt}\left[m\dot{q}_1 + m\dot{q}_2\cos\alpha\right] \Rightarrow$$

$$\Rightarrow g \sin\alpha = \ddot{q}_1 + \ddot{q}_2 \cos\alpha$$

from q_2: $$\ddot{q}_2 = -\frac{m}{m+M}\ddot{q}_1 \cos\alpha \rightarrow$$

$$g \sin\alpha = \ddot{q}_1\left(1 - \frac{m}{m+M}\cos^2\alpha\right) \qquad \ddot{q}_1 = \text{const}.$$

if the slope is of length L: $t = \sqrt{2L/\ddot{q}_1}$