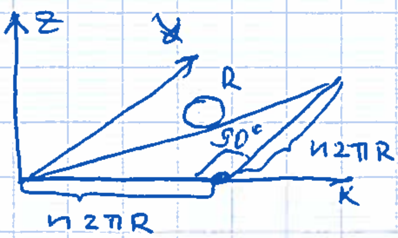
# L32 Holonomic vs non-holonomic systems

**Holonomic:** system has N DOF and can be described with N generalized coordin.

**Non-holonomic:** Number of g. coordinates is different from number of DOF

Example




2 coordinates are insufficient to describe the ball motion.

* Proof of Lagrange equations with constraints

non-constraining Forces: $\vec F = -\nabla U(r,t)$

constraining force: $\vec F_c$

$\vec F_{tot} = \vec F + \vec F_c$ $\quad \mathcal{L} = T - U$ ← does not include const. $F_c$

The action integral:

consider deviation from the right path $\vec r(t)$

$$\vec R = \vec r(t) + \vec \varepsilon(t)$$

$$A = \int_{t_1}^{t_2} \mathcal{L}(\vec R, \dot{\vec R}, t)\, dt$$

$$\delta \mathcal{L} = \mathcal{L}(\vec R, \dot{\vec R}, t) - \mathcal{L}(\vec r, \dot{\vec r}, t) \qquad \mathcal{L}(\vec r, \dot{\vec r}, t) = \frac{m \dot{\vec r}^2}{2} - U(r)$$

$$\delta \mathcal{L} = \frac{1}{2} m \left[ (\dot{\vec r} - \dot{\vec \varepsilon})^2 - \dot{\vec r}^2 \right] - \left[ U(\vec r + \vec \varepsilon) - U(\vec r) \right]$$

$$= m \dot{\vec r} \dot{\vec \varepsilon} - \vec \varepsilon \cdot \vec\nabla U + O(\varepsilon^2)$$

$$\delta A = \int_{t_1}^{t_2} [m \dot{\vec r} \dot{\vec \varepsilon} - \vec \varepsilon \vec\nabla U]\, dt = -\int_{t_1}^{t_2} \vec \varepsilon [m \ddot{\vec r} + \vec\nabla U]\, dt$$

We used: $\int \frac{d}{dt}(m \dot{\vec r} \vec \varepsilon) = \int m \ddot{\vec r} \varepsilon + \int m \dot{\vec r} \dot{\vec \varepsilon} = 0$

$$m \ddot{\vec r} = \vec F + \vec F_c = -\vec\nabla U + \vec F_c \to \delta A = -\int_{t_1}^{t_2} \vec \varepsilon \vec F_c\, dt$$

$$\vec \varepsilon \perp \vec F_c \to \delta A = 0$$