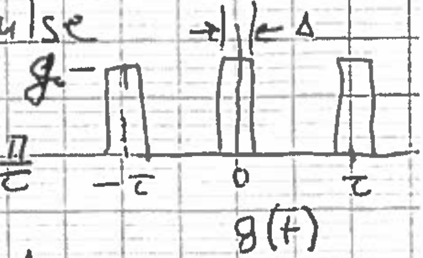
FS of a rectangular pulse



$$g(t) = \frac{a_0}{2} + \sum_n a_n \cos n\omega t \qquad \omega = \frac{2\pi}{\tau}$$

$$a_n = \frac{2 g_0}{n\pi} \sin\left(n\omega \frac{\Delta}{2}\right)$$

$$\frac{a_0}{2} = \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} g(t)\,dt = \frac{g_0 \Delta}{\tau} = g_0 \omega \frac{\Delta}{2\pi}$$

$n \to \infty$ to describe a sharp rect. pulse

$n\omega \to \infty$ — large bandwidth: large frequency $f=$

$n < N_0$: "rectangular" pulse with smooth edges and limited bandwidth: $f < N_0 \omega / 2\pi$

in many cases we may neglect frequencies $f > N_0\omega/2\pi$ and use first few terms ($N \sim$ few) to describe $f(t)$

* Oscillator driven by a rectangular force.

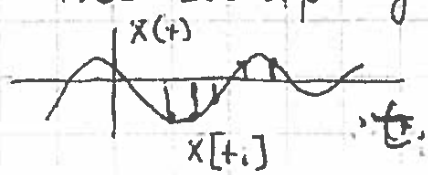
$$A_n^2 = \frac{g_n^2}{(\omega_0^2 - n^2\omega^2)^2 + 4\beta^2 n^2 \omega^2} \sim \frac{g_n^2}{n^4}$$

1) in general contribution of $n \gg 1$ terms can be neglecte

2) Caveat: if $\omega_0 = k\omega$ — resonance at $n=k$

* Nyquist – Shannon theorem (sampling)

a continuous signal $x(t)$ of a finite bandwidth $f < F_0$ can be represented by a sequence of samples $x[t_i]$ if the sampling rate is $\geq 2F_0$



L25 Parseval's theorem (Parseval's Identity)

Oscillator response to a driving force

$$X(t) = \sum_n A_n \cos(n\omega t - \varepsilon_n)$$

in case of a single harmonic
$A = A_{m=n} \neq 0 \quad A_{n\neq m} = 0$ we can characterize the oscillator by its max amplitude A or energy $\sim A^2$: $\langle T\rangle = \langle U\rangle = \frac{1}{2}E = \frac{1}{4}kA^2$

* for multiple harmonics we can use "the average" amplitude calculated as

$$X_{rms} = \sqrt{\langle X^2\rangle} \qquad \langle X^2\rangle = \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} X^2\, dt.$$

rms — root mean square.

* Parseval's Identity.

Use $X(t) = \sum_n A_n \cos(\omega n t - \delta_n)$

can be easily generalized for $X(t)$ that also contains $B_n \sin(\omega n t - \delta_n)$ terms.

$$\langle X^2\rangle = \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} \sum_n \sum_m A_n \cos(n\omega t - \delta_n) A_m \cos(m\omega t - \delta_m)$$

$$\int_{-\tau/2}^{\tau/2} \underset{\sin}{\cos}(n\omega t - \delta_n) \underset{\sin}{\cos}(m\omega t - \delta_m)\, dt = \begin{cases} \tau & n = m = 0 \\ \tau/2 & n = m \neq 0 \\ 0 & n \neq m \end{cases}$$

$$\langle X^2\rangle = A_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2$$

* RMS displacement for a driven oscillator

$$A_n^2 = \frac{f_n^2}{(\omega_0^2 - n^2\omega^2)^2 + 4\beta^2 n^2 \omega^2} \qquad f_n = \frac{2 f_{max}}{\pi n}\sin\left(\frac{\pi n \Delta\tau}{\tau}\right)$$

$\omega = \frac{2\pi}{\tau}$

$\omega_0 = \frac{2\pi}{\tau_0}$