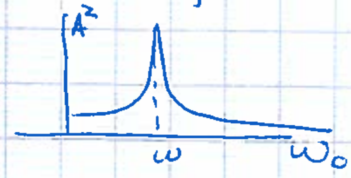
L 23 Resonances

$x(t) = A \cos(\omega t - \delta)$ $\quad A^2 = \dfrac{f_0^2}{(\omega_0^2-\omega^2)^2 + 4\beta^2\omega^2}$

* Very Weak dumping: $\beta \ll \omega_0$



$A_{max} \approx \dfrac{f_0}{2\beta\omega_0}$ Sharp increase of amplitude or resonance @ $\omega_0 = \omega$

* dependence on driving frequency

$\dfrac{dA^2}{dt} = 0 = 4\omega(-\omega_0^2 + \omega^2 + 2\beta^2) = 0 \quad \omega^2 = \omega_0^2 - 2\beta^2$

$A_{max} = \dfrac{f_0}{2\beta}\dfrac{1}{\sqrt{\omega_0^2-\beta^2}} \approx \dfrac{f_0}{2\beta\omega_0}$ for $\beta \ll \omega_0$

* Weak dumping: $\omega^2 > 0$ when $\beta^2 < \dfrac{\omega_0^2}{2}$

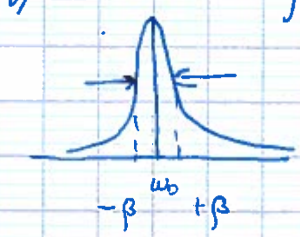
no resonance for $\beta^2 \geq \omega_0^2/2$

\- Width of the Resonance (weak dumping)

FWHM (full width at Half Max)

$A^2(\omega = \omega_0 \pm \Delta) = A^2(\omega = \omega_0)/2 \quad \rightarrow \Delta \sim \beta$

FWHM $= 2\beta$



\- Quality factor (sharpness of the peak)

$Q = \omega_0/2\beta$

$-\tau = 1/\beta$ = decay time $\quad Q = \pi \dfrac{1/\beta}{2\pi/\omega_0} = \pi \dfrac{\text{decay time}}{\text{period}}$

$Q = \pi \cdot (\#\text{ of cycles in } \tau)$

examples: a) Big Ben: $\omega_0 \approx 2\pi \cdot 200\,Hz$. $Q \approx 300$

$\tau \approx 2-3\,sec.$

b) LIGO suspension quartz wires $\quad Q \sim 10^5 \quad f = 512\,Hz$.

$\tau = \dfrac{Q \cdot \text{period}}{\pi} = \dfrac{Q}{\pi f} \sim 100\,sec.$

* The Phase $\quad tg\,\delta = \dfrac{2\beta\omega_0}{\omega_0^2 - \omega^2}$

$\omega \ll \omega_0 \quad \delta \sim 0$

$\omega \approx \omega_0 \quad \delta \sim \frac{\pi}{2}$

$\omega \gg \omega_0 \quad \delta \sim \pi$