

L21

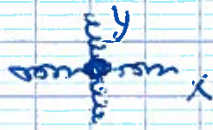
2D oscillators

$$\vec{F} = -k\vec{r}$$

$$F_x = -kx$$

$$F_y = -ky$$

- central force

for small x & y displacement.

$$\ddot{x} = -\frac{k}{m}x$$

$$\ddot{y} = -\frac{k}{m}y$$

$$\omega^2 = \frac{k}{m}$$

* Consider phase-shifted cos solution

$$x(t) = A_x \cos(\omega t - \delta_x)$$

define new

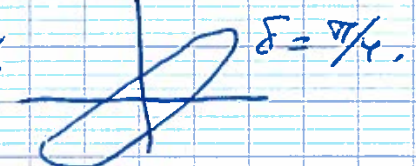
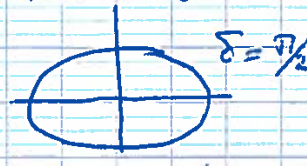
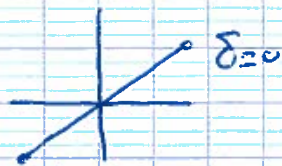
$$y(t) = A_y \cos(\omega t - \delta_y)$$

time $\omega t' = \omega t - \delta_x$

$$\rightarrow x(t) = A_x \cos(\omega t')$$

$$\rightarrow y(t) = A_y \cos(\omega t' - (\delta_y - \delta_x)) = A_y \cos(\omega t' - \delta)$$

x - y oscillator trajectory could be quite different depending on δ :



x

Anisotropic oscillator: $F_x = -k_x x$, $F_y = -k_y y$

$$x(t) = A_x \cos(\omega_x t)$$

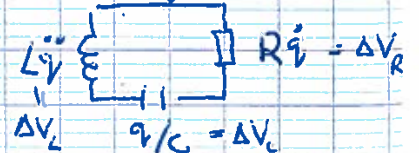
$$y(t) = A_y \cos(\omega_y t - \delta)$$

- $\omega_x/\omega_y = \text{rational} \rightarrow \text{Lissajous figures}$ - $\omega_x/\omega_y = \text{irrational} \rightarrow \text{quasiperiodic motion}$ * Damped oscillations: ^{example} dumping force $-b\dot{v}$

$$m\ddot{x} + b\dot{x} + kx = 0$$

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = 0$$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$$



$$2\beta = b/m \quad \omega_0^2 = \frac{k}{m}$$

homogeneous equation

 β - dumping constant ω_0 - natural frequencyLet's find solution in the form: $x(t) = e^{\Gamma t}$

$$(\Gamma^2 + 2\beta\Gamma + \omega_0^2)e^{\Gamma t} = 0 \quad \text{- auxiliary equation}$$

$$\Gamma_1 = -\beta + \sqrt{\beta^2 - \omega_0^2} \quad \Gamma_2 = -\beta - \sqrt{\beta^2 - \omega_0^2}$$

$$\text{General solution: } x(t) = C_1 e^{\Gamma_1 t} + C_2 e^{\Gamma_2 t}$$

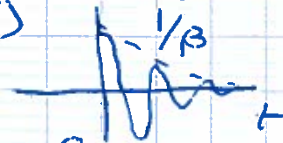
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a) $\beta = 0$ no damping
 $x(t) = C_1 e^{i\omega_0 t} + C_2 e^{-i\omega_0 t}$

b) $\beta < \omega_0$ - weak damping
 $\sqrt{\beta^2 - \omega_0^2} = i\sqrt{\omega_0^2 - \beta^2} = i\omega_d$
 $x(t) = e^{-\beta t} (C_1 e^{i\omega_d t} + C_2 e^{-i\omega_d t})$
 $= A e^{-\beta t} \cos(\omega_d t - \delta)$

eventually
decay
 $e^{-\beta t}$

$1/\beta$ - damping time



c) $\beta > \omega_0$ - strong damping (overdamping)

$x(t) = e^{-\beta t} (C_1 e^{-\alpha t} + C_2 e^{+\alpha t})$ $d = \sqrt{\beta^2 - \omega_0^2}$

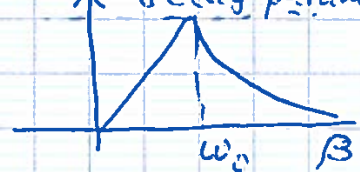
dominant term

d) Critical damping $\beta = \omega_0$ need one more solution: $x_2 = t e^{-\beta t}$

$x(t) = e^{-\beta t} (C_1 + t C_2)$

used for shock absorbers

↑ decay param.



* Driven Damped oscillators

$\ddot{x} + 2\beta \dot{x} + \omega_0^2 x = F(t)/m = f(t)$

$F(t)$ - driving force

* Linear differential operators

$D = \frac{d^2}{dt^2} + 2\beta \frac{d}{dt} + \omega_0^2$ for equation above

Linear: $D(kx) = k D(x)$ $D(x_1 + x_2) = D(x_1) + D(x_2)$

$Dx = 0$ - homogeneous equation

$Dx = f$ - inhomogeneous equation

- particular solution $Dx_p = f$ $D(x_p + \alpha x_h) = f$

- homogeneous solution $Dx_h = 0$