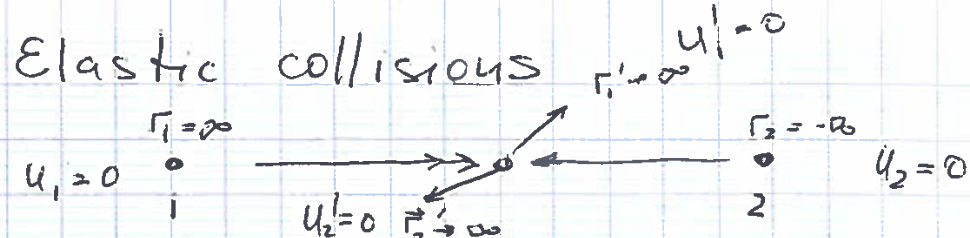
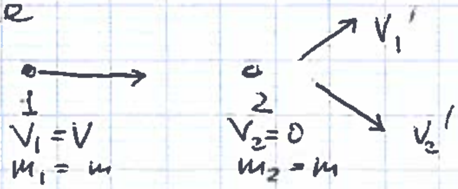
L19 Elastic collisions




Elastic collisions: $T$(before) $= T'$(after)
conservative force

Example 4.8:




$$m\vec{V} = m\vec{V}_1' + m\vec{V}_2'$$

$$\frac{mV^2}{2} = m\frac{V_1'^2}{2} + \frac{mV_2'^2}{2} = (\vec{V}_1' + \vec{V}_2')^2 \frac{m}{2}$$

$$= \frac{mV_1'^2}{2} + m\frac{V_2'^2}{2} + 2m\,\vec{V}_1'\cdot\vec{V}_2' \rightarrow \vec{V}_1'\cdot\vec{V}_2' = 0$$

* Multiparticle systems

pairwise potential energy $U_{\alpha\beta}$

pairwise forces: $\vec{F}_{\alpha\beta} = -\vec{\nabla}_\alpha U_{\alpha\beta}$ $\quad \vec{F}_{\beta\alpha} = -\vec{\nabla}_\beta U_{\alpha\beta}$

$$U_{tot} = \underbrace{\sum_\alpha \sum_{\beta>\alpha} U_{\alpha\beta}}_{\text{potential energy due to interaction between particles (internal forces)}} + \underbrace{\sum_\alpha U_\alpha^{ext}}_{\text{due to external forces}}$$

net force on particle $\alpha$: $\vec{F}_\alpha = -\vec{\nabla}_\alpha U_{tot}$

* Example: build a planet

a) work to add small mass $\Delta m$: $\int_{-\infty}^{R} \frac{GM\Delta m}{r^2}dr = \frac{GM\Delta m}{R}$

change of total potential energy: $\Delta U_{tot} = -\frac{GM\Delta m}{R}$

b) Binding Energy

$$-\int_0^R \Delta U_{tot}(r) = +G\int_0^R \left(\rho\frac{4\pi}{3}\right)^2 \cdot 3r^4 dr = \frac{GM^2}{R}\frac{3}{5}$$

minimum energy to destroy gravitationally bound state.

c) Escape velocity of object: $\Delta U_{tot} + \Delta m\frac{V^2}{2} = 0$

$$V_{escape} = \sqrt{-\frac{2\Delta U_{tot}}{\Delta m}} = \sqrt{2\underbrace{\frac{GM}{R^2}}_{=g}R} \rightarrow \sqrt{2gR}$$

for Earth $V_{escape} \sim 11\,km/sec$

$$T = M_{cm}\frac{V_{cm}^2}{2} + I_{cm}\frac{\omega^2}{2}$$

* Rigig bodies: $\sum_\alpha \sum_{\beta>\alpha} U_{\alpha\beta}(r_\alpha - r_\beta) = const \rightarrow$ ignore.