L15

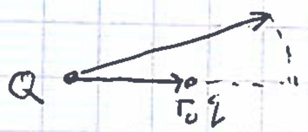
Curl $\vec\nabla \times \vec F = 0$

Example 4.5 $\vec F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^3}\vec r = \frac{\gamma}{r^3}\vec r$

$$(\vec\nabla\times\vec F)_x = \frac{\partial}{\partial y}F_z - \frac{\partial}{\partial z}F_y = \gamma\left[\frac{\partial}{\partial y}\left(\frac{z}{r^3}\right) - \frac{\partial}{\partial z}\left(\frac{y}{r^3}\right)\right] =$$

$$= -\frac{\gamma z\, 3}{r^4}\frac{\partial r}{\partial y} - 3\gamma\frac{y}{r^4}\frac{\partial r}{\partial z} = -\frac{3\gamma}{r^4}(zy - yz) = 0$$

— Potential energy. $U(r) = -\int_{r_0}^{r} F(r)\,dr$



$$U(r) = -\int_{r_0}^{r}\frac{\gamma}{r^2}dr = \frac{\gamma}{r} - \frac{\gamma}{r_0}$$

pick $r_0 \to \infty$ $U(r) = \frac{\gamma}{r}$

$-\vec\nabla U = \vec F$ $F_x = \frac{\partial}{\partial x}\left(\frac{\gamma}{r}\right) = -\frac{\gamma}{r^2}\frac{\partial r}{\partial x} = -\frac{\gamma}{r^3}x$

Time dependent potential energy $U(\vec r, t)$

$\vec\nabla\times\vec F = 0$

$$dU = \vec\nabla U d\vec r + \frac{\partial U}{\partial t}dt = -\vec F d\vec r + \frac{\partial U}{\partial t}dt.$$

$$dT = \vec F d\vec r \to d(T+U) = \frac{\partial U}{\partial t}dt.$$

Energy of one-D system

$U(x) = -\int_{x_0}^{x} F_x(x')\,dx'$ Hook's law $F = -kx$

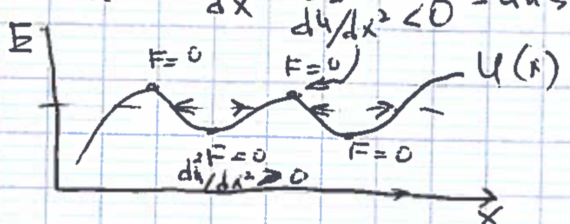
$U(x) = \frac{1}{2}kx^2$ $x_0 = 0$

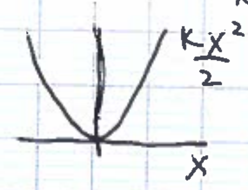
$F_x = -\frac{dU}{dx}$ $d^2U/dx^2 < 0$ - unstable

$U(x)$ min & max.

$\frac{dU}{dx} = 0$

$\frac{d^2U}{dx^2} \geq 0$, < 0





Solution of the motion $T = \frac{1}{2}m\dot x^2 = E - U(x)$

$\dot x = \pm\sqrt{\frac{2}{m}}\sqrt{E - U(x)}$ ← does not work in higher dim.

$$dt = \frac{dx}{\dot x} \to t = \int\frac{dx}{\dot x} = \sqrt{\frac{m}{2}}\int_{x_0}^{x}\frac{dx'}{\sqrt{E - U(x')}}$$

Free fall in fied g. Example 4.6

$U(x) = -mgx$ $\dot x = \sqrt{2gx}$ $(E = 0)$

$t = \sqrt{\frac{2x}{g}}$