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Gradient of $U(\vec{r})$

$$\Delta W (\vec{r} \rightarrow \vec{r} + d\vec{r}) = \vec{F} \cdot d\vec{r} = F_x dx + F_y dy + F_z dz = -[U(\vec{r} + d\vec{r}) - U(\vec{r})]$$

$$= -[U(x+dx, y+dy, z+dz) - U(x, y, z)] = -\left[\frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy + \frac{\partial U}{\partial z} dz\right]$$

$\frac{\partial U}{\partial x}$ - partial derivative

$x, y, z \rightarrow$ independent variables

$$\Delta W = -dU = -\vec{\nabla} U \cdot d\vec{r}$$

$$dU = \vec{\nabla} U \cdot d\vec{r}$$

$$\frac{dU}{d\vec{r}} = \vec{\nabla} U$$

$$\frac{\partial U(x, y, z)}{\partial x} = \frac{\partial U(x)}{\partial x} \quad (y = \text{const}, z = \text{const})$$

$$\vec{F} = -\hat{x} \frac{\partial U}{\partial x} - \hat{y} \frac{\partial U}{\partial y} - \hat{z} \frac{\partial U}{\partial z} = -\vec{\nabla} U$$

operator $\vec{\nabla} = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z}$
gradient

$U(r)$ - gravitational potential energy

Example: $U(r) = -\frac{GMm}{r}$ $r = \sqrt{x^2 + y^2 + z^2}$

$$\frac{\partial U}{\partial x} = \frac{GMm}{r^2} \frac{\partial r}{\partial x} = \frac{GMm}{r^3} x$$

$$\vec{F} = -\frac{GMm}{r^3} (\hat{x} + \hat{y} + \hat{z}) = -\frac{GMm}{r^3} \vec{r}$$

$U(r) = \varphi(r) m$
 $\varphi(r)$ - potential

same for Coulomb force

Curl operator. (Stoke's theorem) problem 4.25

Coulomb field:

$$\varphi(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

$$\vec{F} = -q \vec{\nabla} \varphi$$

line integral $\int_C \vec{F} \cdot d\vec{r}$ does not depend on the path between 1 & 2

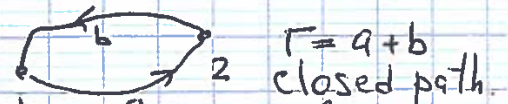
this is equivalent to $\vec{\nabla} \times \vec{F} = 0 \rightarrow$ curl of \vec{F}

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) \hat{x} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) \hat{y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) \hat{z}$$

Problem 4.25

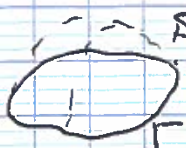
$$\vec{\nabla} \times \vec{F}_q = 0$$

$$\oint_C \vec{F} \cdot d\vec{r} = \int_a \vec{F} \cdot d\vec{r} + \int_b \vec{F} \cdot d\vec{r} = 0$$



closed path for conservative force associated with \vec{F}

We can reverse the problem: given $\vec{\nabla} \times \vec{F} = 0$ we can find $U(\vec{r})$



Stoke's Theorem $\oint_C \vec{F} \cdot d\vec{r} = \iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S}$

$d\vec{S} = |dS| \hat{n}$ - element of the surface S with area $|dS|$ and $\hat{n} \perp$ to the surface.

$$\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S} = 0 \text{ for any surface } S \rightarrow \vec{\nabla} \times \vec{F} = 0$$