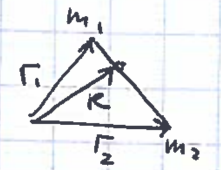
L11 The center of Mass

$$\vec{R} = \frac{1}{\sum m_i} \sum m_i \vec{r}_i \quad \rightarrow (X, Y, Z)$$



2 particles: $\vec{R} = \dfrac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$

- $\vec{R}$ ends on the line connecting m_1 & m_2

$$\vec{r}_1 - \vec{R} = \frac{m_2}{m_1+m_2}(\vec{r}_1 - \vec{r}_2) \qquad \vec{r}_2 - R = -\frac{m_1}{m_1+m_2}(\vec{r}_1 - \vec{r}_2)$$

- $\frac{\partial}{\partial t}\left(\sum m_i \vec{R} = M\vec{R} = \sum m_i \vec{r}_i\right) \rightarrow M\dot{\vec{R}} = \sum m_i \dot{\vec{r}}_i = \vec{P}$

  $\vec{P}$ is total momentum. So

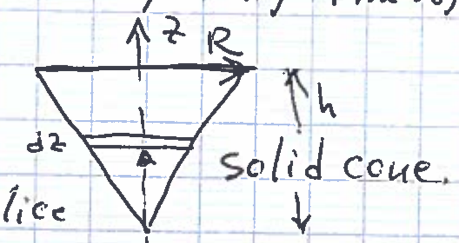
  $F_{ext} = M\ddot{\vec{R}}$

- Lab and CM reference frames (Example with elastic collis.)
- Calculation of CM.

$$R = \frac{1}{M}\sum dm_i \vec{r}_i \Rightarrow \frac{1}{M}\int \vec{r}\, dm = \frac{1}{M}\int \rho \vec{r}\, dV$$

Use different coordinate systems (cartesian, cyl. spherical) depending on the symmetry of the object

- Example 3.2



a) CM should be on z axis

b) mass of the circular slice at z with dz thickness

$$dm = \rho \pi r^2(z)\, dz = \rho \pi \frac{R^2 z^2}{h^2} dz$$

c) CM: $Z = \frac{1}{M}\sum dm_i z_i = \frac{1}{M}\int_0^h z\, dm =$

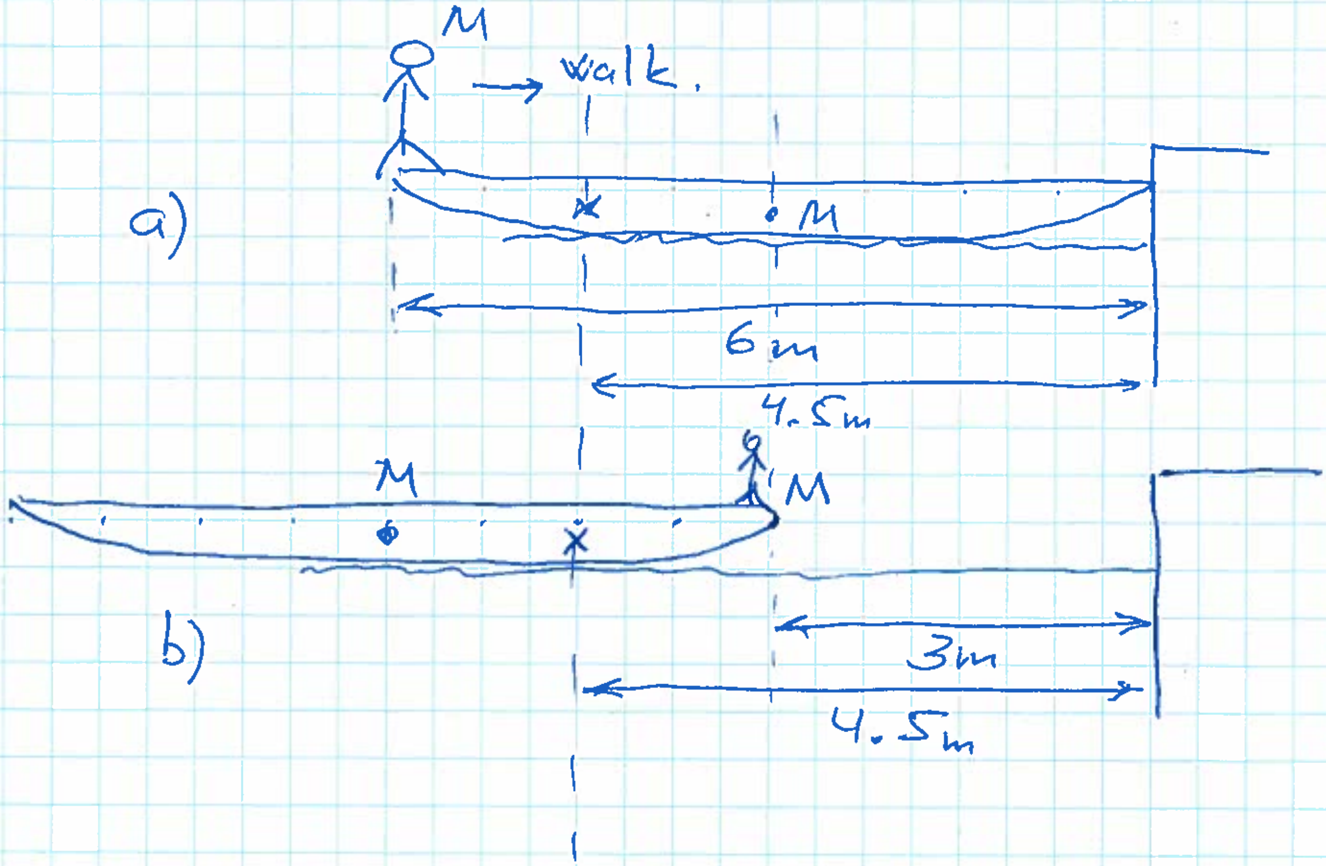
$$= \frac{1}{M}\frac{\rho \pi R^2}{h^2}\int_0^h z^3\, dz = \frac{\rho \pi R^2 h^4}{4Mh^2}$$

d) What is M? $M = \sum dm_i = \int_0^h \frac{\rho \pi R^2}{h^2} z^2 dz =$

$$= \frac{\rho \pi R^2}{3h^2} h^3$$

e) $Z = \frac{3}{4} h$

Problem:

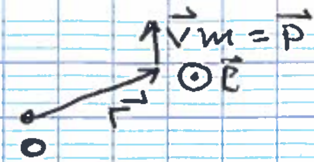


CM did not move

a) Gondolier (mass M) sits on far end of gondola (mass M) and walks (6 m long) toward the pier.
When he/she reaches the other side of gondola, will he/she be able to reach the pier.?

b) No, There are no forces on the center of mass (CM.)
CM will not move
Therefore there will be a gap of 3 m between gondola and the pier.

L11 **Angular momentum.**

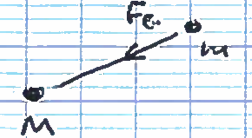


$$\vec{\ell} = \vec{r}\times\vec{p}$$

$$\dot{\vec{\ell}} = \underbrace{(\dot{\vec{r}}\times\vec{p})}_{=0=\dot{\vec{r}}\times\dot{\vec{r}}m} + \vec{r}\times\dot{\vec{p}} = \vec{r}\times\vec{F} = \vec{\Gamma} \text{ - torque}$$

$\dot{\vec{\ell}} = \vec{\Gamma}$ - rotational version of N 2^{nd} law

- Central force (like gravity)



can select origin when $\vec{\Gamma}=0$

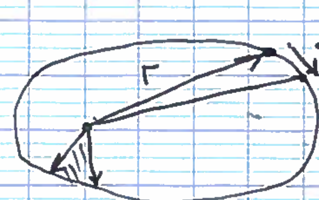
ℓ = const.

conservation of angular momentum

$\vec{r}(t)\times\vec{p}(t)$ defines a plane

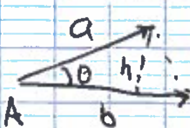
- Kepler's laws - summaries of observed planetary motion (empirical)
  - 1^{st} - planet orbits are ellipses.
  - 2^{nd} - a line segment sweeps equal areas in equal times (dA/dt = const)
  - 3^{rd} - $T^2 \propto a^3$ T - period a - semiMajor a
- Kepler's law can be derived from N's Laws

postpone 1^{st} & 3^{rd} law till later.



$d\vec{r} = \vec{v}\,dt.$

area of a triangle



$a\sin\theta = h$

$\frac{1}{2}|a\times b| = \frac{ab\sin\theta}{2} = A$ $\qquad \frac{bh}{2} = A$

$$\Delta A = \frac{1}{2}[\vec{r}\times\vec{v}\,dt] = \frac{1}{2m}[\vec{r}\times\vec{p}]\,dt$$

$$\frac{dA}{dt} = \frac{1}{2m}[\vec{r}\times\vec{p}] = \vec{\ell}$$