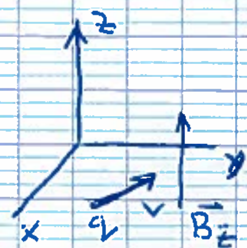
L9

Motion of charge in a Uniform mag field.



$$\vec{F} = q\vec{v}\times\vec{B} = m\dot{\vec{v}}$$

to simplify (no loss of generality)

select $B = (0, 0, B)$, $V = (V_x, V_y, 0)$

$$\vec{V}\times\vec{B} = \det\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ V_x & V_y & 0\\ 0 & 0 & B\end{vmatrix} = (V_y B, -V_x B, 0)$$

$$m\dot{\vec{V}} = q\,\vec{V}\times\vec{B} \rightarrow \begin{array}{l} m\dot{V}_x = qBV_y \rightarrow \dot{V}_x = \omega V_y \\ m\dot{V}_y = -qBV_x \rightarrow \dot{V}_y = -\omega V_x \\ m\dot{V}_z = 0 \end{array}$$

$\omega = \frac{qB}{m}$ cyclotron frequency

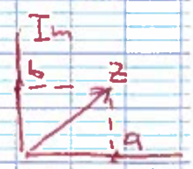
oscillator

Solution 1: $\frac{\partial}{\partial t}(\dot{V}_x = \omega V_y) = \boxed{\ddot{V}_x = \omega\dot{V}_y = -\omega^2 V_x}$

$\frac{\partial}{\partial t}(\dot{V}_y = -\omega V_x) = \boxed{\ddot{V}_y = -\omega\dot{V}_x = -\omega^2 V_y}$

The solutions are $A\sin\omega t$ & $B\cos\omega t$.

Solution 2: $\eta = \underset{Re}{V_x} + i\underset{Im}{V_y}$ $i = \sqrt{-1}$



η is the complex velocity

any complex number $z = \underset{Re}{a} + i\underset{Im}{b}$ can be describe. by a vector

$$\dot{\eta} = \dot{V}_x + i\dot{V}_y = \omega V_y - i\omega V_x$$
$$= -i\omega(V_x + iV_y) = -i\omega\eta$$

two real equations are equivalent to one compx

$$\boxed{\dot{\eta} = -i\omega\eta} \equiv \begin{array}{l} \dot{V}_x = \omega V_y \\ \dot{V}_y = -\omega V_x\end{array}$$

$i\omega = \text{const}$ $\eta = Ce^{-i\omega t}$ = complex solution

L9 Complex exponentials

* $e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} \dots$

$$\frac{\partial e^z}{\partial z} = 0 + 1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} \dots = e^z$$

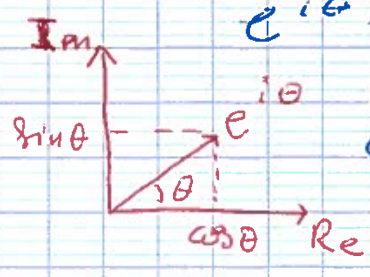
now let $z = i\theta$

$$e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2} \dots = 1 + i\theta - \frac{\theta^2}{2!} + i\frac{\theta^3}{3!} - i\frac{\theta^4}{4!} \dots$$

$$= \left[1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!}\right] + i\left[\theta - \frac{\theta^3}{3!} - \frac{\theta^5}{5!}\right]$$

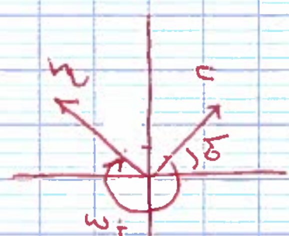
$\cos\theta = 1 - 0 - \frac{\theta^2}{2!} + 0 + \frac{\theta^4}{4!}$ $\quad$ $\sin\theta$

$e^{i\theta} = \cos\theta + i\sin\theta$ Euler's form.



any complex number $z = a + ib$ can be presented as $|z|e^{i\theta} =$
$= |z|(\cos\theta + i\sin\theta)$

$a = |z|\cos\theta$ $\quad$ $|z| = \sqrt{a^2 + b^2}$
$b = |z|\sin\theta$



$$\eta = c e^{-i\omega t} = |c| e^{i\delta} e^{-i\omega t} = |c| e^{-i(\omega t - \delta)}$$

* Solution for charge in magnetic field

angular velocity $\omega = \frac{qB}{m}$

complex coordinate $\zeta = x + iy$ (t)

$\eta = \dot{\xi} = \dot{x} + i\dot{y} = V_x + iV_y$

$\xi = \int \eta(t)\, dt = \int A e^{-i\omega t} dt = -\frac{A}{i\omega} e^{-i\omega t} + const$

Select coordinate system so const=0

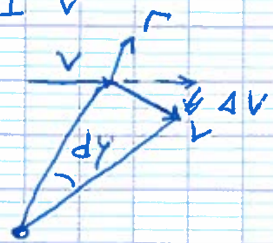
$\xi = \frac{iA}{\omega} e^{-i\omega t} = \frac{A}{\omega} e^{-i\omega t + \pi/2}$

* Larmor radius $r = \frac{v}{\omega} = \frac{mv}{qB} = \frac{p}{qB}$ —

used to measure particle's momentum in High Energy experiments at CERN.

Solution in polar coordinates

$\vec{F} = m\dot{\vec{V}} = q\,\vec{V}\times\vec{B}$ $\quad \vec{F} \perp \vec{V}$



$|V|$ – does not change

in polar coordinates (r, φ)

$$F_r = m(\ddot{r} - r\dot{\varphi}^2) = -qVB$$

$$F_\varphi = m(r\ddot{\varphi} + 2\dot{r}\dot{\varphi}) = 0$$

$$\ddot{r} = 0 \to -mr\dot{\varphi}^2 = -qVB$$

$$\dot{\varphi} = \omega = \frac{V}{r} \to r = \frac{mV}{qB} = \frac{p}{qB}$$ Larmor radius

$$\dot{\varphi} = \omega = \frac{qB}{m}$$ – cyclotron frequency