Air resistance (drag)

- drag $F(v)$ depends on the object Velocity V
  we will consider that $\vec{F}(v) = -f(v)\,\hat{v}$
  Examples when it is not true.
- airplane, spinning ball, sail boat, ...
- consider case when $v \ll S$. (speed of sound)

$\boxed{f(v) = bv + cv^2}$ – Taylor expansion over v/s

$f_{lin} = bV$ – linear (viscous) drag $= \beta D V$

$f_{quad} = cV^2$ – quadratic drag $= \gamma D^2 V^2$

For spherical projectile: $\beta = 1.6\, 10^{-4}\, Ns/m^2$
in STP air $\gamma = 0.25\, Ns^2/m^4$

- Reynolds number: $\boxed{R = Dv\,\rho/\eta}$ ← density (ρ), ← viscosity (η)

  R is a rough approximation for $\frac{f_{quad}}{f_{lin}} = \frac{\gamma}{\beta} DV$

- Example 2.1

  Baseball ($D = 7cm$, $V \sim 5 m/s$) $\frac{f_q}{f_e} \sim 570$

  Millikan oil drop ($D = 1.5\mu m$, $V \sim 10^{-4} m/s$) $\frac{f_q}{f_e} \sim 10^{-7}$

  f_q is dominant when R is large
  f_e is dominant when R is small

- Linear air resistance $\vec{f} = -b\vec{V}$, $m\vec{g}$, x, y

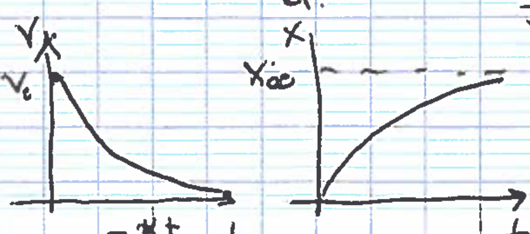
  $\boxed{m\dot{\vec{V}} = m\vec{g} - b\vec{V}}$

  $$\left.\begin{aligned} m\dot{V}_x &= -bV_x \\ m\dot{V}_y &= mg - bV_y \end{aligned}\right\}$$ independent equations for x & y components.

- solution for x: $\dot{V}_x = -\frac{b}{m}V_x = -kV_x$

  or. $\frac{dV_x}{V_x} = -k\,dt \rightarrow \int \frac{dV_x}{V_x} = \int -k\,dt$

  $V_x = V_0 e^{-kt} \leftarrow \ln\frac{V_x}{V_0} = -kt$



$V_0 e^{-kt}$ $\quad$ $V_0/k(1-e^{-kt})$

L6 Solution for y: case a) $\uparrow V_y = V_0$ $\downarrow g$

case b) $\downarrow V_y - V_0$

$$\dot{V}_y = g - \frac{b}{m} V_y$$

regardless how the motion starts (initial cond) the projectile will be falling down and reach its terminal velocity $V_t = \frac{mg}{b}$.

* V_t for small drops

$$b = \beta D \qquad m = \rho \pi D^3/6 \qquad \boxed{V_t = \frac{\rho \pi D^2}{6\beta} g}$$

$$\beta = 1.6 \cdot 10^{-4} \qquad V_{ter} = \frac{840 \cdot \pi (1.5 \cdot 10^{-6})^2}{6 (1.6 \cdot 10^{-4})} \cdot 9.8 \approx 6 \cdot 10^{-5}\ m/s.$$

case b)

$$\dot{V}_y = \frac{b}{m}(V_t - V_y) = -\frac{b}{m}(V_y - V_t)$$

$$u = V_y - V_t \qquad \frac{du}{dt} = \frac{dV_y}{dt} = \dot{V}_y = \dot{u} \rightarrow \dot{u} = -\frac{b}{m} u$$

same equation as for X case: $u = A e^{-kt}$

$$V_y - V_t = (V_{y0} - V_t)\, e^{-kt}$$ defined A from initial conditions

$$V_y(t) = V_{y0} e^{-kt} + V_t (1 - e^{-kt})$$

$$V_y(\infty) = V_t.$$

* characteristic time $V_t = g/k = g\tau$

$$V_{y0} = 0 \longrightarrow V_y(t) = V_t (1 - e^{-t/\tau})$$

in $t = \tau$ a falling object reaches 63% of V_t

or τ is time when object reaches V_t if accelerated with const $a = g$.

case a) Case a) $y\uparrow$ $\uparrow V_{y0}$ reverse y axis $\to V_t \to -V_t$

$$V_y(t) = V_{y0} e^{-t/\tau} - V_t (1 - e^{-t/\tau})$$

$$V_y(0) = V_{y0} \qquad y(t) = \int_0^t V_y\, dt = -V_t t + (V_{y0} + V_t)\tau \times (1 - e^{-t/\tau})$$

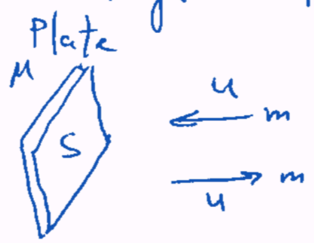
$$V_y(\infty) = -V_t$$

projectile reaches max elevation

$$y_{max} = y(t_m) \quad \text{when} \quad V_y(t_m) = 0$$

L06

Origin of the quadratic drag

a) 

$M >> m$

$\Delta P = 2um$ – momentum transfer from particle m to the plate.

Δt – interaction time.

$\frac{\Delta P}{\Delta t} = F$ force on the plate

b) 

plate moving in ideal gas

n – number of particles per unit of volume

u – velocity of the plate

S = area of the plate.

$\Delta P = mu$ – momentum transfer from a single particle.

$nuSdt$ – number of interacting particles in time Δt.

$$\frac{\Delta P_I}{\Delta t} = \frac{\Delta P}{\Delta t} \cdot nuS. = mnSu^2$$

Quadratic drag force
for objects of different shape

$$F_{quad} = \frac{1}{2} C\rho S u^2$$

Drag Coefficient Values
Typical values of drag coefficient C.

Object	C
Airfoil	0.05
Toyota Camry	0.28
Ford Focus	0.32
Honda Civic	0.36
Ferrari Testarossa	0.37
Dodge Ram pickup	0.43
Sphere	0.45
Hummer H2 SUV	0.64
Skydiver (feet first)	0.70
Bicycle	0.90
Skydiver (horizontal)	1.0
Circular flat plate	1.12