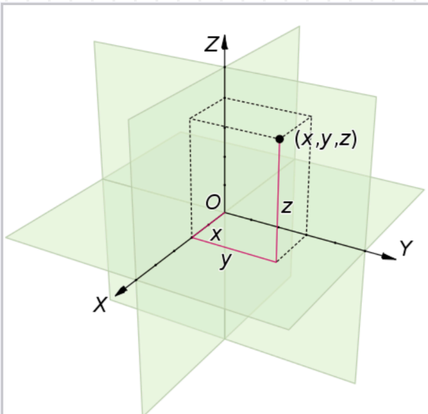
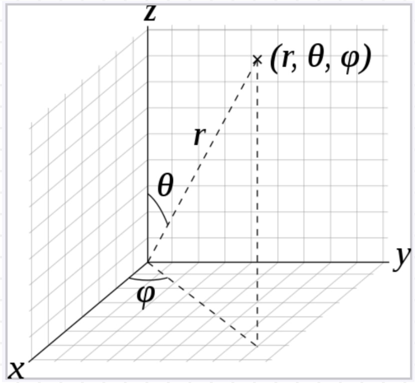
L05

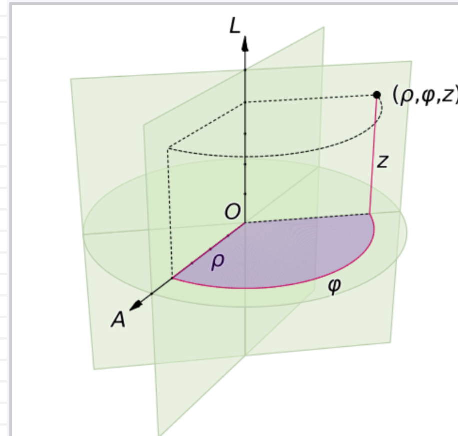
(x,y,z) cartesian



3D(r,θ,φ) spherical
2D(r,φ) polar



(ρ,φ,z) cylindrical



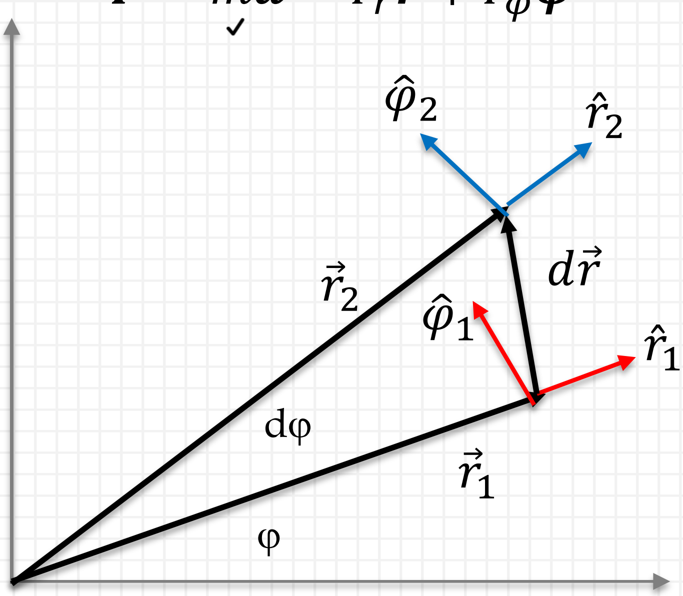
transformation to cartesian coordinates

$$x = r\sin\theta\cos\varphi$$
$$y = r\sin\theta\sin\varphi$$
$$z = r\cos\theta$$

$$x = \rho\cos\varphi$$
$$y = \rho\sin\varphi$$
$$z = z$$

Second Newton's law in 2D polar coordinates

$$\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}} = F_r\hat{\boldsymbol{r}} + F_\varphi\hat{\boldsymbol{\varphi}}$$



$$d\vec{r} = \vec{r}_2 - \vec{r}_1 = \vec{r}(t+dt) - \vec{r}(t)$$

$$\dot{\boldsymbol{r}} = \frac{d}{dt}(r\hat{r}) = \dot{r}\hat{\boldsymbol{r}} + r\dot{\hat{\boldsymbol{r}}}$$

$$\dot{\hat{\boldsymbol{r}}} = \dot{\varphi}\hat{\boldsymbol{\varphi}} \text{ (prove)}$$

$$\ddot{\vec{\boldsymbol{r}}} = \frac{d}{dt}(\dot{r}\hat{\boldsymbol{r}} + r\dot{\varphi}\hat{\boldsymbol{\varphi}})$$

$$\ddot{\vec{\boldsymbol{r}}} = \ddot{r}\hat{\boldsymbol{r}} + \dot{r}\dot{\hat{\boldsymbol{r}}} + \dot{r}\dot{\varphi}\hat{\boldsymbol{\varphi}} + r\ddot{\varphi}\hat{\boldsymbol{\varphi}} + r\dot{\varphi}\dot{\hat{\boldsymbol{\varphi}}}$$

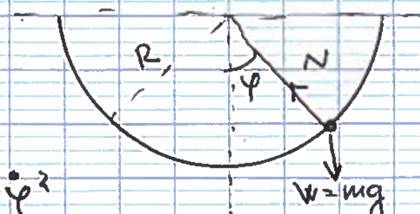
$$\dot{\hat{\boldsymbol{\varphi}}} = -\dot{\varphi}\hat{\boldsymbol{r}} \text{ (prove)}$$

$$\ddot{\vec{\boldsymbol{r}}} = (\ddot{r} - \underset{\text{centripetal}}{r\dot{\varphi}^2})\hat{\boldsymbol{r}} + (r\ddot{\varphi} + \underset{\text{Coriolis}}{2\dot{r}\dot{\varphi}})\hat{\boldsymbol{\varphi}}$$

2nd Newton's law in polar coordinates

$$\boldsymbol{F_r} = m\,(\ddot{r} - r\dot{\varphi}^2)$$
$$\boldsymbol{F_\varphi} = m\,(r\ddot{\varphi} + 2\dot{r}\dot{\varphi})$$

L5 Example 1.2 An Oscillating Skateboard.



$r = R$

$F_r = -mR\dot{\varphi}^2$ $\qquad F_r = mg\cos\varphi - N$

$F_\varphi = mR\ddot{\varphi}$ $\qquad = \qquad$ $F_\varphi = -mg\sin\varphi$

$$-mg\sin\varphi = mR\ddot{\varphi}$$

$$\ddot{\varphi} = -g/R \sin\varphi$$

$$\ddot{\varphi} \approx -g/R\, \varphi \qquad \varphi \ll 1.$$

define $\omega = \sqrt{g/R}$ $\quad \ddot{\varphi} = -\omega^2\varphi$

solutions: $\varphi_1(t) = A\sin\omega t$

$\varphi_2(t) = B\cos\omega t$

$\varphi(t) = \varphi_1(t) + \varphi_2(t)$ – general solution.

- consider initial conditions.
- period of oscillations: $\tau = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$