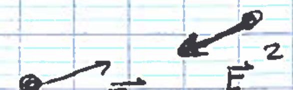
L4

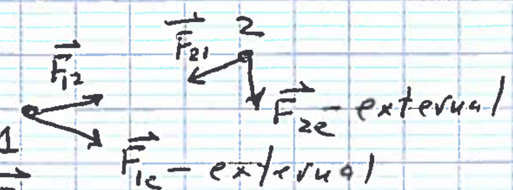
The third Newton's law



$$\vec F_{12} = -\vec F_{21}$$

- Conservation of ~~t~~ total momentum

* 2 particles



$$\vec F_1 = \vec F_{12} + \vec F_{1e}$$
$$\vec F_2 = \vec F_{21} + \vec F_{2e}$$

write second law as: $\dot{\vec P}_1 = \vec F_1 \quad \dot{\vec P}_2 = \vec F_2$

total momentum $\vec P = \vec P_1 + \vec P_2$

total momentum rate of change: $\dot{\vec P} = \dot{\vec P}_1 + \dot{\vec P}_2$

$$\dot{\vec P} = \underbrace{\vec F_{12} + \vec F_{21}}_{=0} + \vec F_{1e} + \vec F_{2e} = \vec F_e$$

if $\vec F_e = 0 \quad \dot{\vec P} = 0$ – conse~~p~~rvation of t

* many particles

$$\vec F_\alpha = \sum_{\beta \neq \alpha} F_{\alpha\beta} + \vec F_{\alpha e}$$

$$\dot{\vec P} = \sum \dot{\vec P}_\alpha = \sum_\alpha \sum_{\beta \neq \alpha} \vec F_{\alpha\beta} + \sum_\alpha \vec F_{\alpha e}$$

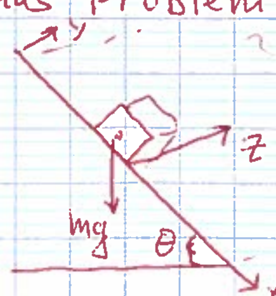
contains $N \cdot (N-1)$ terms → always ev

$$\sum_\alpha \sum_{\beta \neq \alpha} \vec F_{\alpha\beta} = \sum_\alpha \sum_{\beta > \alpha} \vec F_{\alpha\beta} + \vec F_{\beta\alpha} = \emptyset$$

* Principle of Conservation of Momentum

if $\vec F_{ext} = 0 \qquad P = const.$

- Bonus Problem [20 pts]



$\mu = tg\,\alpha$ – the friction coeffic

at $t=0$ we push block

along Z direction with

velocity $u = V_z(t=0)$

What is the direction and

magnitude of velocity $\vec V(t \to \infty)$

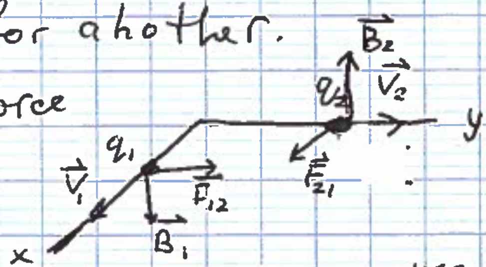
L4

Validity of the third law.

in the domain of CM all 3 Newton's law are considered to be "exact" — valid with some accuracy.

* For $v \sim c$ they break down
For example, the 3d Law assumes
$$\vec{F}_{12}(t) = -\vec{F}_{21}(t)$$ at the same moment of time t
it could be true for one observer, but false for another.

* "non-central" force



$\vec{F}_{12} \sim \vec{V}_1 \times \vec{B}_1$ $\quad$ $\vec{F}_{21} = \vec{V}_2 \times \vec{B}_2$ $\quad$ use "right corkscrew" rule

* Problem 1.32

- electric field at position 1 due to charge 2
$$\vec{E}(\vec{r}_1) = \frac{1}{4\pi\varepsilon_0} \frac{q_2}{|\vec{r}_1 - \vec{r}_2|^2} \vec{e}_{\vec{r}_1 - \vec{r}_2}$$

ε_0 – permitivity

- magnetic field
$$\vec{B}(\vec{r}_1) = -\frac{\mu_0}{4\pi} \cdot \frac{q_2}{|\vec{r}_1 - \vec{r}_2|^2} [\vec{e}_{\vec{r}_2 - \vec{r}_1} \times V_2]$$

μ_0 → permeability

$$\vec{F}_{12\,el} = q_1 \vec{E}_1(\vec{r}_1) \qquad \vec{F}_{12\,B} = q_1 [V_1 \times \vec{B}(\vec{r}_1)]$$

assuming that $\vec{V}_1 \perp \vec{V}_2 \rightarrow \vec{V}_1 \perp \vec{B}(\vec{r}_1)$

$$|F_{12E}| = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \qquad |\vec{F}_{12B}| = \frac{\mu_0}{4\pi} \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} V_1 V_2$$

$$\frac{|\vec{F}_{12E}|}{|F_{12B}|} = \frac{V_1 V_2}{c^2} \ll 1$$ we can ignore the violation of the 3d law

$\frac{V^2}{c} \sim 1 \longrightarrow$ need relativistic mechanics

Force & momentum

Second Newton's Law :

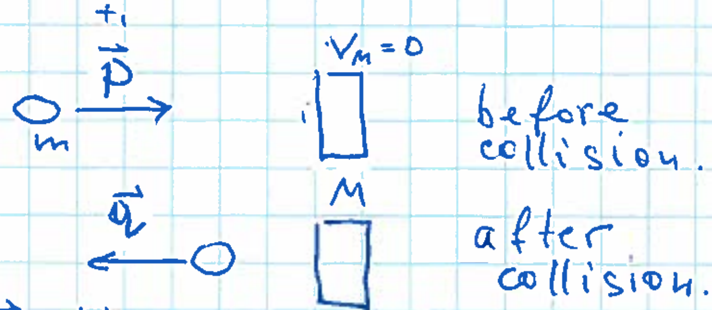
a) $\vec{F} = m\vec{a}$ $\quad$ $p = mv$ $\quad$ $\dot{p} = ma$

b) $\vec{F} = \dot{\vec{p}}$ $\leftarrow$ not equivalent in relativity

external force is changing particle's momentum $\quad \vec{F} = \frac{d\vec{p}}{dt} \rightarrow d\vec{p} = \vec{F}dt.$

or $\quad \Delta p = \int_{t_1}^{t_2} \vec{F}\, dt$

Example a :



$\Delta P_m = \vec{q} - \vec{p} = \int \vec{F}_{Mm}\, dt$ $\quad$ $\vec{F}_{Mm}$ – force acting from mass M to m

no external forces for a system of M & m

total momentum before :

total momentum after :

$\vec{p} = \vec{Q} + \vec{q} \rightarrow \vec{q} - \vec{p} = -\vec{Q}$

$\vec{Q} = \int \vec{F}_{mM}\, dt = -\int \vec{F}_{Mm}\, dt = -(\vec{q} - \vec{p})$