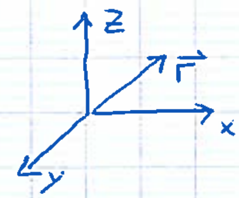
L2

* **Cartesian coordinates**



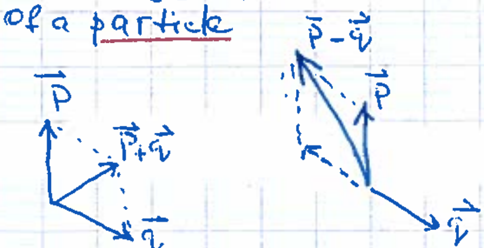
- describe a position of a pointlike body with
- radius vector $\vec r$
- notations: $\mathbf{r}$, $\vec r = x\hat x + y\hat y + z\hat z = (x, y, z)$

$\hat x, \hat y, \hat z \to$ unit vectors along cartesian axes: or $\vec e_x, \vec e_y, \vec e_z$

- coordinate index: $r_1 = x$, $r_2 = y$, $r_3 = z$

$\vec r = \sum_i r_i \vec e_i = \sum_{i=1}^{3} r_i \vec e_i$ – describe position of a particle

* **Vector operations**



- addition, subtraction:
  $\vec p \pm \vec q = (p_1 \pm q_1, p_2 \pm q_2, p_3 \pm q_3)$

$\vec p \cdot \vec q = \sum_i p_i q_i = |p||q| \cdot \cos\theta$

- Scalar (dot) product: $\vec p \cdot \vec q = (\vec p \cdot \vec q) = \sum_i p_i q_i$
- Vector length (norm): $\sqrt{\vec p \cdot \vec p} = |\vec p|$

  use Pythagora's theorem in cartesian coordinates.

  $\vec p \cdot \vec q = |p||q|\cos\theta$ → θ – angle between vectors $\vec p$ and $\vec q$

- Vector product (cross product)

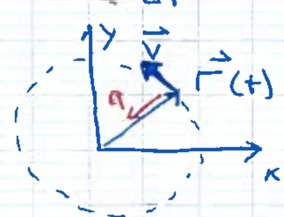
$$\vec p \times \vec q = \begin{vmatrix} \vec e_1 & \vec e_2 & \vec e_3 \\ p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \end{vmatrix}$$

* **differentiation of vectors** $\frac{d\vec r}{dt} = \lim_{\Delta t \to 0} \frac{\vec r(t+\Delta t) - \vec r(t)}{\Delta t}$

  velocity $\vec v = \frac{d\vec r}{dt} = \dot{\vec r}$

  acceleration $\vec a = \frac{d\vec v}{dt} = \dot{\vec v} = \ddot{\vec r} = \frac{\partial^2 \vec r}{dt^2}$

* **Example:** particle on circular orbit.



$\vec r(t) = (R\cos\omega t, R\sin\omega t)$

$|\vec r| = R = \text{constant}.$

$\vec v = \dot{\vec r} = (-\omega R\sin\omega t, \omega R\cos\omega t)$

$|\vec v| = \omega R$ $\quad \vec v \cdot \vec r = 0 \to \vec v \perp \vec r$

$\vec v \cdot \vec r = \omega R^2(-\sin\omega t, \cos\omega t) \cdot (\cos\omega t, \sin\omega t) =$

$= \omega R^2(-\sin\omega t\cos\omega t + \cos\omega t\sin\omega t) = 0$

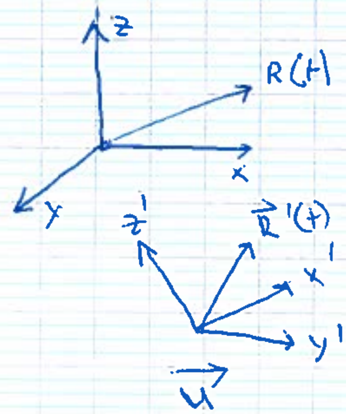
$\vec a = \dot{\vec v} = R\omega^2(-\cos\omega t, \sin\omega t)$; $|\vec a| = R\omega^2$

check that $\vec v \cdot \vec a = 0$

L2

- Reference frames: in physics we have a freedom to select the coordinate system, the origin of time and how the coordinate system moves

* transformation between frames.



(a) Change origin of time
$\vec{R}'(t) = \vec{R}(t+t_0)$

(b) Shift – change spatial origin $\vec{R}'(t) = \vec{R}(t) + \vec{R}_0$

(c) rotation – change orientation of axes

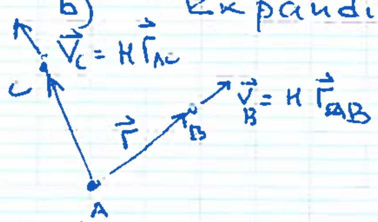
$$(x', y', z') = \begin{vmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{vmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

(d) moving with const speed $\vec{u}$: $\vec{R}'(t) = \vec{R}(t) + \vec{u}t$

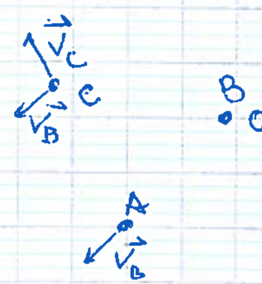
- * Our perception of the world can be very different depending on the choice of the ref. fra[me]

a) N. Copernicus (1543) On the Revolution of Cel. s[pheres]

b) Expanding universe:



change $\rightarrow$ frame from A → B



Is galaxy A in the center of the univerce?? (?)

$$\vec{V}_C' = \vec{V}_C - \vec{V}_B = H(\vec{R}_{AC} - \vec{R}_{AB}) = H\vec{R}_{BC}$$

an observer in the galaxy B sees that galaxies around him/her move away

- Inertial & non-inertial reference frames
Sections 1.2 & 1.3