Exam 3 Solutions

1. This is Newton's Third Law! The two forces are equal and opposite

2. Momentum is conserved. Therefore $|m_a v_a| = |m_b v_b|$

   So $|v_b| = (3/2)|v_a|$ (not surprising that b goes faster)

   $E_a/E_b = m_a v_a^2/m_b v_b^2 = (3/2)*(2/3)^2 = 2/3$

3. We can use the simple kinematic equation that $\mathbf{v} = \mathbf{v_0} + \mathbf{a}t$ and take up as positive.
   $\mathbf{v} = +2.0 - 9.8*1 = -7.8$ m/s. so speed is 7.8 m/s.

4. Average velocity is displacement/time. Displacement is distance in a straight line between beginning and ending points. We see that the two legs of the journey make the two sides of a (5,12,13) right-angle triangle, so the average velocity, $|v| = 130/3 = 43$ km/hr.

5. This is the same function as was given in an earlier test, but the question is different. If you sketch the function you find that it has a minimum between x=0 and x=-2. It is when U is a minimum then v is a maximum. To find this location, differentiate U and set dU/dx = 0.
   $4x + 4 = 0$, and so $x = -1$. So it goes from $x = 0$ to $x = -1$ to reach its maximum speed.

6. The KE for each is the sum of the translational KE and the rotational KE. Clearly the one that is the slowest is the one which has the highest fraction of its energy as rotational KE. That is the thin annular cylinder (see face-sheet of test), and the one with the highest is the one with the lowest fraction of its energy as rotational KE, and that is sphere.

7. We add up the Rotational Inertia from the left-most rod, [$(1/12)ML^2$] the second rod (0 rotational inertia) and the circle [$(1/2)ML^2$] and that is the answer.

8. If we take the torque around point B, we know that the counter-clockwise torque equals to clockwise torque. Therefore $(0.80*Mg) + (0.5*5g) = (1.0*30)$
   $M=(30-2.5*9.8)/(0.8*9.8) = 0.7$ kg

9. As U(infinity) = 0, and we are happy to have K at infinity to be zero, that makes 0 mechanical energy at infinity. Hence all we need is zero mechanical energy at the surface. (This means that the kinetic energy is GMm/R, but that is not the question). This was a homework question.

10. Newton's second Law of motion together with Newton's Law of Gravity.

    $GMm/R^2 = mv^2/R$ and so $v^2= GM/R$
    The period is given by $T = 2\pi R/v$, and so $T^2 = 4\pi^2R^3/GM$ and $M = 4\pi^2R^3/GT^2$
    Now we can plug numbers in.
    $M = 4*9.86*(10^5)^3/(6.67*10^{-11}*(27*24*60*60)^2)$ all in SI units.
    $= 4*9.86*10^{35}/(6.67*5.441*10^{12})$

$= 6.3*10^{16}$ kg

11. The gravitational force needs to be sufficient to make the material on the surface accelerate in a circle.
    $GMm/R^2 = mv^2/R$
    As above: $M=4*9.86*(20*1000)^2/(6.67*10^{-11}*1)$
    $M=5*10^{24}$ kg

12. The pressure is proportional to the depth, so that on average, the pressure over the whole area is the pressure at a point half way down (17.5m from the top). We take that pressure and multiply by the area.

    Force = Pressure x Area = $(1000*9.8*17.5)*(35*314) = 1.9*10^9$ Newtons

13. We know from Archimedes Principle that to float with it all submerged means that the weight of the object must equal the weight of the fluid displaced.
    So, keeping in grams and centimeters for convenience:
    $\rho_w(4\pi/3)r_o^3 = \rho_s(4\pi/3)*(r_o^3-r_i^3)$

    $1.00*60^3 = 7.87*(60^3-r_i^3)$
    $(60^3-r_i^3) = 27445$
    $r_i = 57$ cm (which could be guessed…..)

14. There are many ways of doing this. If we know the velocity of the block immediately after the bullet hits, we can then use energy conservation to find the displacement. To find the velocity immediately after it hits, we can use momentum conservation.
    630*0.0095 = 5.4v, so v = 1.108 m/s
    $0.5*5.4*1.108^2 = 0.5kx_{max}^2$ and $x_{max} = 0.033$ m = 3.3 cm

15. The laws of physics do not depend on the velocity, so clearly answers b) c) and d) are not correct. They do change if you are in a frame that is accelerating. As the period is less, looking at the formula for a pendulum we can see that the effective acceleration due to gravity has to be greater than g. This occurs when the elevator is accelerating upwards.

16. We know that (it's in the formulae), for a physical pendulum, $\omega^2 = hmg/I$. Here, $I = (1/3)mL^2$, and h = 0.5 (the distance from half-way along the rule to the pivot point). That gives $\omega^2 = 1.5g$, where g is the acceleration due to gravity.

The period, $T=2*\pi/\omega$. So $0.85=6.283/\omega$ and $\omega=6.283/0.85= 7.39$ rad/s and so from $\omega^2 = 1.5g$, "g" = 36.4 m/s^2

17. We know the formula that relates the speed with the Tension and the linear mass density. To find the speed, and we know the speed from the wave equation, because speed = ω/k where k is the term that multiplies x and ω the term that multiplies t.
    So we have v = 12/4.7 = sqrt(T/mu) and mu = 3.83 kg/m

18. We know that speed = frequency x wavelength. Here the wavelength with the lowest frequency is the one where half a wavelength fits in the wire, so therefore λ = 20 meters.
    V = sqrt(T/mu) = sqrt(250/0.01), because mu = 0.1/10 (mass divided by length)
    So 20 f = sqrt(250/0.01) and f = 7.9 Hz

19. Let's think of what the truck perceives. It is moving in the opposite direction to the sound. So using the formula we have $v_{detector}$ = -45 m/s, and so f (in kHz) = 150*(345+45)/345 = 1.69 kHz. It then emits 169 kHz back. Now we have source moving with + 45 m/s and the detector not moving, so f = 169*(345/(345-45)) = 195 kHz. Note that three of the answers defy common sense.
20. One of the waves has to go L further than the other. We need them to be exactly out of phase and that is characterized by $\lambda/2$, so that means that the L = $\lambda/2$