1. Two small identical metal spheres carry charges of μC and 8.5 μC and are 5.66 m apart. Now the spheres are touched together and again separated to 5.66 m. What force does one exert on the other? (in N)

(1) 1.36×10^{-3} (2) 3.34×10^{-3} (3) 15.67×10^{-3} (4) 6.07×10^{-4} (5) 10.38×10^{-2}

Solution:

Once the two sphere touch, the charge is homogenously distributed through the two spheres.
Each one will have a charge of (8.5–4.1)/2 = 2.2μC.
Now

$$\begin{aligned} F &= k\frac{q_1 q_2}{r_{12}^2} && (1)\\ &= 8.99\times10^9\frac{\mathrm{Nm}^2}{\mathrm{C}^2}(\frac{2.2\times10^{-6}\mathrm{C}}{5.66\mathrm{m}})^2 && (2)\\ &= 8.99(\frac{2.2}{5.66})^2\times10^{-3}\mathrm{N} && (3)\\ &= 1.36\times10^{-3}\mathrm{N} && (4)\end{aligned}$$

Thus, the answer is (1).

2. Two point charges are on the x-axis. A 3.6 μC charge is at x = 0 m and a -0.4 μC charge is at x = 6 m. Where can a third charge be placed so that the net force on it is zero? (in m)

(1) x = 9 (2) x = 2 (3) x = −5 (4) x = 4 (5) x = 12

Solution:

When is the net force zero? When the force from the first F_1 compensates the force from the second F_2 charge.

$$\begin{aligned} F_1 &= k\frac{3.6\times10^{-6}q_3}{(0-x)^2}\frac{\mathrm{C}}{\mathrm{m}^2} && (5)\\ F_2 &= -k\frac{0.4\times10^{-6}q_3}{(6-x)^2}\frac{\mathrm{C}}{\mathrm{m}^2} && (6)\end{aligned}$$

Now

$$\begin{aligned} F_1+F_2 &= 0 && (7)\\ k\frac{3.6\times10^{-6}q_3}{(0-x)^2}\frac{\mathrm{C}}{\mathrm{m}^2} &= k\frac{0.4\times10^{-6}q_3}{(6-x)^2}\frac{\mathrm{C}}{\mathrm{m}^2} && (8)\\ \frac{3.6}{(0-x)^2} &= \frac{0.4}{(6-x)^2} && (9)\\ 3.6(6^2-12x+x^2) &= 0.4x^2 && (10)\\ 6^2-12x+x^2 &= \frac{x^2}{9} && (11)\\ 6^2-12x+\frac{x^2}{9} &= 0 && (12)\end{aligned}$$

using the standart formula for a quadratic equation

$$x_{1/2} = \frac{9}{8}\frac{12 \pm \sqrt{(12)^2 - 4 \times 6^2\frac{8}{9}}}{2} \quad (13)$$
$$= \frac{9}{8}(6 \pm 2\sqrt{9-8}) \quad (14)$$
$$= \frac{9}{8}(6 \pm 2) \quad (15)$$

so that $x_1 = 9$ and $x_2 = \frac{9}{2}$. Thus, the answer is (1).

3. A uniform electric field, with a magnitude of 1625 N/C, is directed parallel to the positive x-axis. If a proton is released from rest at x = - 4.0 m, what is its kinectic energy as the proton reaches x = 4m? (in joules)
(1) 2.08×10^{-15} (2) 9.6×10^{-16} (3) 1300 (4) 0 (5) 3.84×10^{-14}

Solution:
There is a constant force $F = 1625\frac{\text{N}}{\text{C}} \times 1.6 \times 10^{-19}\text{C} = 1.625 \times 1.6 \times 10^{-16}\text{N}$. Now $E = F \times 8\text{m} = 20.8 \times 10^{-16}\text{Nm}$.
Thus, the answer is (1).

4. Two charges are placed on the x axis. One is 2.0 μC at x = -1 m. The other is 3.0 μC at x = 2.0 m. What is the electric field at the point x = 0 due to these charges? (in N/C)
(1) 1.13×10^2 (2) 3.34×10^2 (3) 15.67×10^2 (4) 6.07×10^4 (5) 10.38×10^2

Solution:
A charge of q_0 at x_0 creates an electric field at r of

$$E(x_0 - r) = k\frac{q_0}{|r - x_0|^2} \quad (16)$$

Charge 1 creates thus a field at 0m of $E_1 = k\frac{2.0\times10^{-6}\text{C}}{\text{m}^2}$ while charge 2 creates a field of $E_2 = k\frac{3.0\times10^{-6}\text{C}}{4\text{m}^2}$ that we have to add to compute the total field. At $x = 0$m is

$$E(0\text{m}) = 10^{-6}\text{C}k\frac{\text{Nm}^2}{\text{C}^2}(\frac{2.0}{\text{m}^2} + \frac{3.0}{4\text{m}^2}) \quad (17)$$
$$= 8.99 \times 10^3\frac{\text{N}}{\text{C}}(2 + \frac{3}{4}) \quad (18)$$
$$= 2.47 \times 10^4\frac{\text{N}}{\text{C}} \quad (19)$$

Thus answer (4) is closest.

5. Adding the values according to the rules $R_t = R_1 + R_2$ when in series, or $\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2}$ when in parallel. Looking at the figure we find

$$\begin{aligned}
R_t &= \{(3.0\Omega + 5.0\Omega)^{-1} + [(\frac{1}{6\Omega} + \frac{1}{12\Omega})^{-1} + 4.0\Omega]^{-1}\}^{-1} && (20)\\
&= \{(8.0\Omega)^{-1} + [\frac{12\Omega}{3} + 4.0\Omega]^{-1}\}^{-1} && (21)\\
&= \{\frac{1}{8.0\Omega} + \frac{1}{8.0\Omega}\}^{-1} && (22)\\
&= 4.0\Omega && (23)
\end{aligned}$$

Thus answer (1) is correct.

6. We use the formula $V = I \times R$ (Ohm's law), so the total current is

$$\frac{12\text{V}}{4.0\Omega} = 3\text{A} \quad (24)$$

Since the equivalent resistors up and down are the same according to 5.), we immediately see answer (1) is correct.

7. Wire 1: $R_1 = 8\Omega$ for an area A_1 of the wire. Doubeling the diameter $d_2 = 2d_1$ means squaring the area the current can flow through because

$$A_1 = \frac{\pi}{4} d_1^2 \Rightarrow A_2 = 4A_1 \quad (25)$$

On the other hand the length is 3 times the original one.
Thus, we solve

$$R_t = 3(\frac{1}{R_1} + \frac{1}{R_1} + \frac{1}{R_1} + \frac{1}{R_1})^{-1} = \frac{3}{4} R_1 = 6\Omega \quad (26)$$

Again, answer (1) is correct.

8. We calculate the circut on the right using that the sum of all voltages present in a loop must be zero. Thus, together with the first Kirchhoff law

$$I_1 = I_2 + I_3 \quad (27)$$

in the left loop

$$\begin{aligned}
5\text{V} + 8\text{V} - 3\text{V} + V_{1u} + V_m &= 0 && (28)\\
I_1 R_1 + I_3 R_m &= 0 && (29)\\
10\text{V} + 10\Omega I_1 + 10\Omega I_3 &= 0 && (30)
\end{aligned}$$

Furthermore, the right loop is

$$5\text{V} + 10\Omega I_3 - 10\Omega I_2 = 0 \quad (31)$$

Moreover, the full loop is by analogy

$$5\text{V} + 10\Omega I_1 + 10\Omega I_2 = 0 \tag{32}$$

Thus, Eq. (1) is missing and incorrect.

9. We go through the list:

1. Same emf, two lightbulbs in parallel ⇒ each has the same voltage. Thus, they shine just as bright as X.
2. (Correction:) Three emf, two lightbulbs in series ⇒ each has one half of the tripled voltage (3/2). Thus, they shine brighter as X.
3. For the same reason as above does each lightbulb have 2/3 of the voltage applied. Thus, they do not shine as bright as X.
4. As before, but now each has 1/2 of the voltage applied. Thus, they do not shine as bright as X.
5. Two emf's in parallel, so their voltage is the same as just one emf. However, there are two lightbulbs in series ⇒ each has 1/2 of the voltage applied. Thus, they do not shine as bright as X.

10. The charge on a capacitor is

$$Q = C \times V \tag{33}$$

First, we need calculate what the equivalent capacitor is, i.e. how much charge is in fact in the system. The rule is

$$\begin{aligned} C_t &= [C_1^{-1} + (C_2 + C_3)^{-1}]^{-1} & (34) \\ &= [\frac{1}{4\mu\text{F}} + \frac{1}{16\mu\text{F}}]^{-1} & (35) \\ &= \frac{16}{5}\mu\text{F} & (36) \end{aligned}$$

Thus, with 20V applied, there is a charge of

$$Q_t = 20 \times \frac{16}{5}\mu\text{C} = 4^3\mu\text{C} \tag{37}$$

All capacitors in series have the same charge $Q_1 = Q_2 + Q_3 \equiv Q_t$. Since C_2 and C_3 are parallel, they have to have the same voltage

$$\begin{aligned} V_2 &= V_3 & (38) \\ \frac{Q_2}{C_2} &= \frac{Q_3}{C_3} & (39) \\ \frac{Q_2}{Q_3} &= \frac{C_2}{C_3} = \frac{10}{6} & (40) \end{aligned}$$

together with

$$\begin{aligned}
4^3\mu\text{C} &= Q_2 + Q_3 && (41)\\
\frac{4^3\mu\text{C}}{Q_3} &= 1 + \frac{Q_2}{Q_3} && (42)\\
\frac{4^3\mu\text{C}}{Q_3} &= 1 + \frac{10}{6} && (43)\\
16 \times 4\mu\text{C} &= \frac{16}{6} Q_3 && (44)
\end{aligned}$$

Thus, we find

$$Q_3 = 24\mu\text{C} \qquad (45)$$

which is answer (1).