

Lecture W2  
**Motion of an Object and Newton's Law**

### 1. Motion of an Object

Anything in motion will change its position in time. *Here, I am simplifying the meaning of motion because a spinning top is in motion sometimes without changing its position, and a boomerang can fly and at the same time rotate. To be more precise, we have to include the rotation of an object. However, in this class we will assume that there is no spin of an object.* Therefore, to be able to describe motion of an object quantitatively, we need to know the position at every moment.

**Concept of center of mass** and replacing an object with a mathematical point. Then where should we put the representative point? **Demo of center of mass of Florida.**

Suppose  $x$  is the position at a certain time  $t$ . We can express this information in a very simple way without using a long sentence:  $x(t)$ . This simple expression perfectly replaces the sentence. For example, I am driving to Orlando on I-75. Let's mark Exit 390 (39th Ave exit) as the origin to measure the distance from. I can record the time when I pass every exit. Since I know the exit mile, I can list many quantities,  $x_1(t_1)$ ,  $x_2(t_2)$ ,  $x_3(t_3)$ ,  $x_4(t_4)$ , ... Here,  $x_i(t_i)$  means after time  $t_i$  from the departure time I pass Exit  $x_i$ .

Here are the recordings. Forgive me. I will use conventional units only for this example. What kind of information I can get out of these recordings (measurements)? What is the

TABLE I:

	1	2	3	4	5	6	7	8	9
$x_i$ (mile)	3	6	21	35	46	61	89	95	117
$t_i$ (min)	2.6	5.0	19.2	32.5	41.7	56.0	78.0	85.8	100.3

total time of driving? What is the total distance of the trip? Let's get more serious! What is the average speed? For the third question,

Average speed is 70.0 mph. Now you can calculate the speed in each section (between two recordings). There is no guarantee that I used cruise control in each section and drove at a constant speed. But your calculation will give an average speed in a specific section.

**Q1** Is there any section where I violated the speed limit?

In physics when you plot your measurements on a graph properly, most of the time it is much easier to get the required information and a clear understanding.  
(PPT)

You see that the slope of the straight line connecting two points is the average speed in that section. When you connect the origin and the last point with the straight line, its slope is 70.0 mph. It is now clear where I sped up and slowed down.

### **Displacement and Distance**

In the above example, the distance is plotted on a straight axis. I-75 is far from a straight line. So the plot was done as if the curved road is straightened without stretching or contracting. There are two different lengths you can think of. One is the distance discussed above. The other is the length of the straight line connecting the Exit 390 to Orlando Exit (on the map), which should be shorter than the distance mentioned. The latter is called *Displacement*. You may think this difference is created because the road is not straight. *Not true!* Consider a straight road. Start from the origin (zero displacement point), move 150 m in one direction. At this point the distance and the displacement are the same, 150 m. However, if you turn around move backward by 30 m, the distance of travel is 180 m but the displacement is 120 m. So pay attention what quantity is used in a certain expression. You know well speed  $v$  is calculated by

$$v = \frac{\text{Distance of travel}}{\text{Time of travel}} = \frac{\Delta x}{\Delta t}.$$

Here, Greek letter  $\Delta A$  indicates change of the quantity  $A$ :  $\Delta A = (\text{final value of } A) - (\text{initial value of } A)$ .

**Distance:** length along the path of travel.

**Displacement:** the shortest length and direction from the origin to the destination.

**Example:** An object is moving on a straight line. The object was at  $x(2) = 3.5$  m (at the position 3.5 m from the origin at time 2 s) and reached  $x(7) = 13.5$  m (at the position 13.5 m at time 7 s). What are  $\Delta x$  and  $\Delta t$ ? Calculate the average speed.

**Q2** One month of diet produced a change in mass ( $m$ ) (I did not use weight here),  $\Delta m = 3.2$  kg. If the mass after diet is 76.5 kg, what was the mass before diet? Did the diet program work?

## 2. Newton's Law of Motion

Imagine that you are in a spaceship cruising through the space at a constant speed. How would you describe the motion of your spaceship. From your point of view, the spaceship does not move because you are sitting in a passenger seat throughout the journey. If your spaceship (seat) was moving relative to yourself, you would be out of the seat. However, from the control tower on Earth you and the spaceship are definitely moving at a certain fixed speed. The important point here is that *exactly the same motion can be described differently depending on the view point*. In physics the view point is called the *reference frame*.

Let us do this thought experiment ("gedanken experiment"). Hang an orange from the ceiling in a dark room with a spot light on it. The orange is stationary as described by  $v = 0$  at a specified location, say  $x(t) = 0$  for any  $t$ . If you take a video with your smart phone at a fixed angle of pointing while you are walking smoothly in one direction with a constant speed. When you replay your video, you will see the orange moving backward smoothly at the same speed. Again, in essence the stationary state of the orange is equivalent to the orange moving on a straight line at a constant speed. This seemingly trivial fact is extremely important in understanding physics of motion, which cost many hard years of brilliant physicists. In other words, *physics should be exactly the same when things happen*

on Earth or in a spaceship or a reference frame moving on a straight line at a constant speed—we call this uniform motion from now on. If you can juggle three balls in this classroom, you should be able to juggle exactly the same way in a smoothly moving train. We also know from our experience that if you do not push or pull, an object at rest remains still. Applying the argument we just made, we logically conclude that an object moving at a constant speed on a straight path does not experience any force on it. Then what is the consequence of force in the motion of an object?

### **Newton's 1<sup>st</sup> Law of Motion**

*Every object continues in its state of rest or equivalently uniform motion unless it is compelled to change that state by a force impressed on the object: Galileo's Principle of Inertia*

This means a change in motion of an object (in speed and direction of motion) is caused by a force acting on the object. Newton's 2nd law describes how a force affects the motion of an object in a quantitative manner in the form of mathematical equation.

### **Newton's 2<sup>nd</sup> Law of Motion**

*The change in motion, the rate of change of direction and speed ( $a$ ), is proportional to the force ( $F$ ) on the object, and is made in the direction in which force is impressed. The proportional constant is mass of the object  $m$ .*

$$F = ma.$$

This is the famous Newton's equation of motion where  $a$  is the acceleration:  $a = \frac{\Delta v}{\Delta t}$ . The first and second laws are compacted in this single equation. The power of this simple equation cannot be overemphasized. Once you know the information of force, you can predict the motion in response to the force without ambiguity. Inversely, if you have detailed information of motion, you may extract the nature of the relevant force. Here is how it works.

1. An object of mass 1 kg is sitting on a very smooth surface. The surface is so smooth that the object moves without any resistance or friction. When the object is pushed horizontally with a force of 1 N (check the SI unit of force is newton), the object starts to move in the same direction of the force, the speed starts to increase with the rate given by the equation:

$$a = \frac{\Delta v}{\Delta t} = \frac{F}{m} = \frac{1\text{N}}{1\text{kg}} = 1 \text{ N/kg}.$$

Since ... So, as far as the force is applied the speed will increase with this rate. At a

certain point the force is removed. Then you see  $a = 0$  and  $\Delta v = 0$ , meaning that the speed remains constant at the value at the point of force removal.

2. It is possible that there are multiple forces acting on an object. In this case the force in the equation should be the *total net force*. Since force is a vector quantity with magnitude and direction, it is possible that two forces act on a body but completely cancel each other. Then the total net force in the equation should be 0, in effect as if no force was applied.

3. *Change in the direction or speed in a motion is caused by a force. If there is no change in the direction or speed of an object, then there is no net force acting on the object.*

### **Newton's 3<sup>rd</sup> Law of Motion**

*To every force there is always opposed an equal force called reaction force; when two bodies interact, the force acting on body 1 by the other body 2 is equal and opposite to the force acting on body 2 by body 1.*

Demo of reaction cart.

### **3. Falling Body and Constant Circular Motion**

Put your self in deep space far away from any stars or planets. You are holding an apple in your hand. Open your hand gently detach your hand from the apple. The apple will stay stationary at the position of your release. This means that there is no net force acting on your apple. Now back to Earth Galileo found all objects fell at a same rate regardless of their mass. That means there must be a net force (gravity) acting on the object downwards. Galileo also confirmed that the speed of a falling body increases at a constant rate,  $\frac{\Delta v}{\Delta t} = \text{const}$ . Invoking the definition of acceleration and Newton's law, the only way to produce the findings is with a force proportional to the mass of the object. For the falling object on Earth, that acceleration is  $9.8 \text{ m/s}^2$  and we call it  $g$ . For most of the purpose we can use  $g = 10 \text{ m/s}^2$ . Gravity seems to be a quite unusual force because the force acts on a body without pushing or pulling by contact. So in old days people thought there were invisible angels or demons attached to the body pushing the object.

**Examples:** An object of 3 kg is released 50 m above the ground. What would be the speed of the object 1, 2, and 3 s after the release?

**Example:** An object of 1 kg is released 100 m above the ground. What would be the speed of the object 1, 2, and 3 s after the release?

**Q3** Suppose an object is moving on a straight path with a constant speed of 9.8 m/s. For the duration of 3 s, the object traveled  $9.8 \times 3 \approx 30$  m. If an object falls from a high tower for 3 s, did it travel longer or shorter than 30 m? Can you calculate the exact distance of fall? How would you estimate the distance of fall? *Consider the average speed.*

Consider a motion of an object circling around a center point at a constant speed. Is there a net force acting on the object? Why?

Now we know that a circular motion is caused by a force that pulls the object towards the center of the circular path. The planetary motion seems to be a circular motion. You may imagine Earth circling around the sun, which means there is a force pulling Earth towards the sun, or the sun is pulling remotely Earth to itself. According to the 3rd law, Earth pulls the sun with the same strength of force to itself. Then why the sun does not circling around Earth?