PHY1033C **Fall 2017**

Lecture W3

Measurement Uncertainty and Law of Gravitation

1. Measurement Uncertainty

Can you make a measurement of a quantity absolutely free of uncertainty? Even with utmost care, it is inevitable to have a certain level of error. However, as our understanding of nature gets deeper and more sophisticated, more accurate measurements are warranted. For this scientists and engineers have to design measurement systems with higher resolution and precision, but it is also very important to understand the source of the uncertainty and figure out how to minimize it.

Suppose you measure your weight with a scale. The LED panel displays your weight to the level of 0.1 lb. You step on the scale and read 120.4 lb, and repeat the measurements multiple times and write down all those measurements. Then your record would look like the one in the table below.

TABLE I:

	1	2	3	4	5	6	7	8	9
weight (lb)	120.4	120.4	120.3	120.5	120.2	120.6	120.4	120.3	120.5

There are a few things you may want to consider here:

1. You have no idea what would be the value at a finer level than 0.1 lb. The reading 120.4 could be any value between 120.35 and 120.44. So it generates ± 0.05 lb uncertainty around the value 120.4. The fluctuations in the measurements also provides an uncertainty of 0.4 lb ranging from 120.2 to 120.6.

2. What causes the fluctuation of the measurements between 120.2 and 120.6.

3. How do you know that the scale that you used is accurate? If you use a different scale, it may have produced different numbers because the scale is out of calibration.

Some of the uncertainty is caused directly by the limitation of measuring instruments. So no matter how carefully you conduct measurement and no matter who conduct the measurements, that amount of uncertainty cannot be eliminated. This type of error that affects all measurements the same way is called *systematic error*. The only way to reduce is to better design of the measurement system. The rest belongs to *random error*. One clear way to reduce random error is to make multiple measurements and use the average value. This is a consequence of statistical nature of the measurement. For example, the measurement in the above example varies around the average weight, 120.4 lb. Statistical analysis can prove that a statistical error decreases as you increase the number of measurements (N): $Error \approx \frac{1}{\sqrt{N}}$. So for $N = 9$ (above example), the uncertainty is about 0.33 (33%), not very reliable, while for $N = 1000$, it reduces to 0.03 (3%). Polling is a kind of measurement: measure individual opinion for many people. Therefore, it carries error or uncertainty associated with it. During an election season, you hear the results of numerous polls. Many of those poll uses about 1000 people pool, which gives about 3% uncertainty. So, when you do your experiment, make as many as measurements and take the average value as your final result.

2. Law of Gravitation

Nicolaus Corpernicus (1473 - 1543), Tycho Brahe (1546 - 1601), Galileo Galilei (1564 - 1642), and Johannes Kepler (1571 - 1630) are a group of scientists who devoted their lives to understand the heavenly motion of planets. Brahe with his vigor and persistence made detailed measurements of planetary motion for many years. Kepler, who was a talented mathematician with a analytic mind, worked as an assistant for Brahe in his late years. Somehow he inherited Brahe's massive data and went though painful procedures of analysis. Kepler knew that the orbit of Earth around the sun was elliptic with the sun at its focal point rather than a perfect circular path. Kepler already argued that the gravity was a universal attraction between the objects. His thoughts and understanding of the planetary motion are summarized in three laws:

1. The orbit of the planets is an ellipse with the sun at one focus.
2. The area swept out by a line drawn from the sun to a planet is the same in equal time interval.
3. The square of the length of each planet's year (period of the planet orbit, T) is proportional to the cube of the long axis of the orbit (L): $T^2 = AL^3$ or $T = BL^{3/2}$.

Exercise A communication satellite circles around once a day at 6.8 Earth radii. Knowing that the Moon orbits at 60 Earth radii, estimate the period to complete the orbit. Which law should we apply?

Newton linked the motion of an an object on Earth to the heavenly planetary motion in the sky. He went further to establish a concrete mathematical form for the gravity between two objects, law of gravitation: a specific form of force between two massive object:

$$F = G\frac{mM}{r^2}.$$

Here, m and M are the masses of two objects, G is the fundamental gravitational constant ($G = 6.67^{-11}$ $\mathrm{m^3kg^{-1}s^{-2}}$, and finally r is the distance between two objects. *So far, you learned two fundamental physical constants: c and G. There will be two more coming.* This specific form of the gravity is consistent with the Kepler's law. Between any two objects with non-zero mass will feel attractive force and the strength of the force is proportional to the product of the masses and weakens as the distance gets larger.

• Gravity is always attractive. You will learn that Coulomb force between two charges has identical form but can be attractive or repulsive depending on the charge types.

• Mass m is pulled towards mass M. Mass M is also pulled by mass m with the same strength. They form an action-reaction pair (Newton's 3^{rd} law).

Suppose there are two objects with mass m and M and assume that M is much larger than m. This is pretty similar to the pair of the sun (M) and Earth (m) or Earth (M) and a satellite orbiting around Earth (m): $M \gg m$ for both cases. The sun pulls Earth with the strength given by the Newton's law and at the same time Earth also pulls the sun with the same strength, F_G (here I put a subscript G to indicate the gravity). Since there is a force on acting each object, it will cause change in direction and speed of each object. As a result, Earth orbits around the sun. But what happened to the sun?

Q1 Two lead balls are hanging freely suspended by thin strings from the ceiling. The mass of the ball is 30 and 500 kg each. they are separated by 0.01 m. *Here, we consider the gravitational force between two lead balls only. The gravity due to Earth is completely canceled by the string. That is why the balls are not falling down vertically.*
(1) What is the strength of gravity on each ball due to the other lead ball?
(2) Since there is a force acting on each ball, it will start to move with an acceleration. What is the acceleration of each ball? Should they the same?

3. Linking Gravitation to Falling Body

The force on a falling object of mass m is gravity between the object and Earth. Galileo measured the acceleration $g = 10$ m/s^2. Therefore as Newton did we can link gravity to the falling object by

$$F = gm = G\frac{mM_{Earth}}{R_{Earth}^2},$$

where M_{Earth} and R_{Earth} are the mass and radius of Earth, respectively. Therefore, we can get $g = G\frac{M_{Earth}}{R_{Earth}^2}$. Since we can measure g on Earth, knowledge of either G or M_{Earth} will allow us to measure the other quantity. Newton did not know the value of G although it is a fundamental constant in his theory. About 150 years later, Henry Cavendish (1731 - 1810) performed a brilliant experiment to measure G with 1% accuracy.