Lecture W7

Electric and Magnetic Fields

1. Force and Field

When you apply a force on an object by pushing or pulling, you can clearly see the action of force because you make a direct contact on the object. However, gravitational or Coulomb force acts at a distance without a direct physical contact. How does the force actually act though an empty space? How do they know how far are the objects separated to produce the right strength consistent with the law inversely proportional to the square of separation? How do they know each others mass or charge? And does the force act immediately without any delay in time?

A transformative idea was concocted by Michael Faraday based on his laborious and ingenious experiments and keen observations. Faraday was born in 1791 about a year after the death of Benjamin Franklin. Throughout his life, he received almost no formal education and was self-taught like Franklin. He worked in his teenage as an apprentice to a local bookbinder. He made enormous contribution to science, chemistry and physics, and changed the way we live now. Although he did not know mathematics which is considered essential skills for any scientists, he made meticulous records on his experiments and observations.

Faraday introduced the concept of a field which revolutionized the understanding the nature of the forces acting at a distance–long-ranged forces. He wanted to concertize the abstract nature into a substantial entity. I believed that the force at a distance acts through a field which fills the space. As a visualization, he introduced (force) field lines as shown in the figure. Each charge is a source or sink of a field. As you can see, a certain number of field lines are attached to a point charge extending radially and infinitely. The types of charge is determined by the direction of the field lines: outward (inward)lines for a positive (negative) charge. You may say that a positive (negative) charge is a source (sink) of a field. *The absolute number of lines is not important since field lines are merely a way of visualization. But the relative number of lines is crucial.* For example, you can draw 10 or 100 lines for an electron with a charge $e = -1.6 \times 10^{-19}$ C. But once you set the number, a charge with $2e$ should have twice the number of lines.

What would happen when two charges are nearby? The field lines from different sources

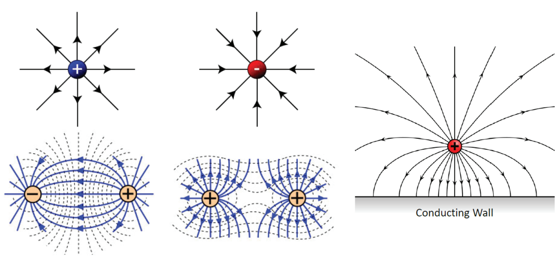

FIG. 1:

interact and are modified from the lines of an isolated charge. But the number of lines from each charge should not change because the amount of charge does not change. This will naturally result in unique configuration of the lines for a given pair of charges. For two opposite charges, the field lines can satisfy the local constraints rather easily. Lines which come out of a positive charge enter into the negative charge allowing for smooth bending of the lines. On the other hand, two positive charges have a conflict. To avoid this conflict the field lines in the conflicting region repel each other (see Fig. 1). These pictures will give you an intuitive sense for attraction or repulsion. This picture should coalesce into Coulomb force. For this, a new quantity, electric field strength (E) is defined as

$$E = \frac{F_C}{q} = k\frac{Q}{r^2} \quad \text{or} \quad F_C = qE.$$

This concept is quite convenient because now we can associate Coulomb force between two charges with a physical quantity of a single charge. Let's use E to describe Coulomb force from a point of view of a single charge. Place a charge Q at the origin. This charge will emanate electric filed lines throughout the space as shown in the figure. The strength of this field line is proportional to the amount of charge of itself and weakens inversely proportional to the square of the distance from the charge. When a second charge of q is placed at a distance r from the first one, it will feel the electric field strength $E = k\frac{Q}{r^2}$ which results in Coulomb force $F_C = qE$ with its own charge q.

The right figure in Fig. 1 shows an interesting situation in which a single positive charge is placed near a conducting wall. A rigorous calculation gives you the field lines as shown. The field lines come out of the positive charge radially but end at the conducting surface (making the right angle with the conductor). Reflecting the field lines between two opposite charges, you can make two arguments: 1.There must be (negative or positive) charges on the wall. 2. Therefore, there must be (attractive or repulsive) force between them. Yes, when a negative (positive) charge is placed near a conductor, the will be positive (negative) charge appears on the surface of the conductor because Coulomb force repeals the negative (positive) charge away from the surface resulting in positive (negative) charge on the surface.

2. Gravitational Force and Coulomb Force

As mentioned earlier, the mathematical structure of two forces are identical. So we can establish a specific connection between them. As you can see in the table, the electric field has exactly the same mathematical structure as the gravitational acceleration. Therefore, we can also visualize gravitational field lines (gravitational acceleration) as lines emanating from a mass. When another mass is placed in space, the mass will feel a force due to the gravitational field in space.

TABLE I: Comparison between Electrostatics and Gravity

	Constant	Source	Force	Field	Potential E.
Electrostatics	k	charge (+ or -)	$k\frac{qQ}{r^2} = qE$	$E = k\frac{Q}{r^2}$	qEd*
Gravity	G	m (only +)	$G\frac{mM}{r^2} = mg$	$g = G\frac{M}{r^2}$	mgh

*Ed is called electric potential, commonly known as **volt**.

3. Magnetic Field

Magnetism is arguably the oldest subject in physics. Ancient Greeks and Chinese realized certain strange stones (permanent magnets) attracted irons. About 1000 years ago Chinese already used a compass for navigation. In his book, *On the Magnet and Magnetic Bodies, and on the Great Magnet the Earth* published in 1600, William Gilbert proposed that the Earth itself was a gigantic magnet. For a long time, people knew only one source of magnetism from iron. In 1821, a Danish physicist, Oersted noticed that an electrical wire carrying current made the nearby compass reorient in a specific direction. This was the first clue for an inter-relation between electricity and magnetism.

magnetic field lines around a wire and the right-hand rule)

Faraday thought the reorientation of the compass around a current-carrying wire must have caused by a force which was quite different from the known Coulomb or gravitational force. Somehow the force was directed in a winding fashion rather than along direction between two objects. Electrical current is simply moving charge (electrons). Moving charge results in a changing electric field. A changing electric field caused a force on a compass. Therefore, there must be a force field created by the wire in a circular fashion represented as a magnetic field (B). Faraday was bothered by a profound question: Why does only moving charge produce a magnetic field? If moving charge (electric current) can produce a magnetic field, can a varying magnetic field can produce an electric field? Through a series of brilliant experiments, Faraday answered this question: electricity and magnetism are reciprocal. In other words, a changing electric field produces a magnetic field and vice versa! This was a ground-breaking realization which has made enormous impact on our everyday life. Based on this, Faraday demonstrated that he could generate electricity by moving a magnet in a copper coil, the first demonstration of modern generator. The principle to produce electricity by a changing magnetic field is called Faraday induction. Your bike may have a gadget which uses this principle.

A constant current is produced by electrons collectively moving at a constant speed. We have already discussed that a constant current produces a static magnetic field in the direction dictated by the right-hand rule. If the electors move with an acceleration, for example, oscillate, then the magnetic field induced by the changing electric filed is no longer static and varies in time. And then this changing magnetic field acts back to induce an electric field, and so on. Therefore, it is no longer meaningful to think about electric and magnetic fields as separate entities and now we simply call electro-magnetic field. Fluctuating electro-magnetic field is nothing but *light*. Depending on high fast it fluctuates, it could be microwave, visible light, x-ray, or gamma ray in the order of increasing frequency.

There is another interesting aspect. A magnetic field exerts a force only on a charge in motion. If a charge is stationary, no force from a magnetic field. If you place a copper wire in a static magnetic field, the wire does not feel any force because copper is not a magnetic material (meaning does not get attracted by a permanent magnet). However, if you flow a current trough the wire, a force appears in a specific direction determined by the relative direction of the current and the magnetic field. You can set this direction using another right-hand rule. The force is proportional to current. This means for a single charge the force is proportional to the speed of the charge and amount of charge. This force is called Lorentz force:

$$F_L = qvB \quad \text{for a sing charge of q,}$$

$$F_L = BIL \quad \text{for a wire of lengh } L \text{ with current } I$$

Figure 2 below shows how the right-hand rules work. Magnetic filed comes out of the north pole and enters into the south pole.

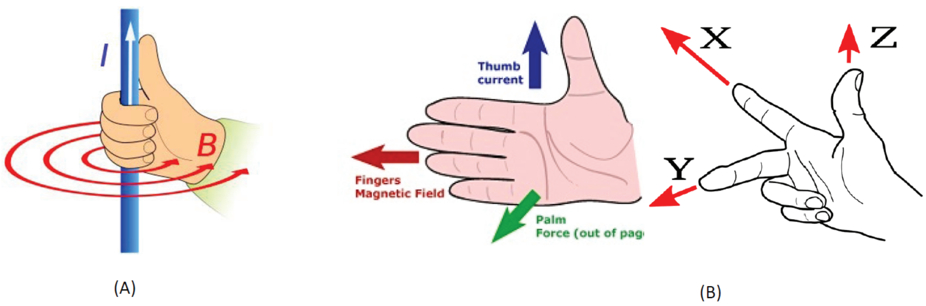

FIG. 2: (A) The right-hand rule for induced magnetic field by a current carrying wire. (B) The right-hand rule for force on a current carrying wire by a magnetic field. Two equivalent right-hand rules: X: current; Y: magnetic field; Z: force.