Lecture W5

Absoluet Zero and Ideal Gas Law

1. Absolute Zero

A gas at temperature T is composed of a large number of gas atoms or molecules each of whom carries kinetic energy equivalent to $k_B T$. When you raise the temperature of a gas, a certain amount of thermal energy is transferred to gas through heat. The absorbed heat (energy) is used to increase the speed of molecules to reach the average speed of the molecules given by $k_B T \approx m v_{av}^2$. Suppose that the gas container is on a heater. The heat from the heater will raise the temperature of the bottom. When molecules collide with the bottom, they absorb energy and reflected back into the gas with a higher speed. The high speed molecules now make multiple collisions with other (slower) molecules and transfer energy to them. Through this many many scattering, all molecules in the container pick up their speed. This is how the whole gas get warmer although the heat is supplied from the bottom surface. This process is quite efficient and fast. To give a sense, in air each gas molecule will make about a billion collisions with other gas molecules in one second! I encourage you to visit http://www.falstad.com/gas/ and play with the simulation. Click the **Heater** button and vary the **Heater Temperature**. It might help to slow down the **Simulation Speed**. The heater is at the bottom and pay attention to the changes near the bottom and how the changes propagate through the volume.

Since kinetic energy, $\frac{1}{2}mv^2$, cannot be negative, it sets the lowest temperature, $T = 0$ in kelvin scale. This is called *Absolute Zero*. In other words, at absolute zero, the gas molecules stop to move. One can expand this concept to liquids and solids. In solids the constituent atoms or molecules vibrate around their fixed positions rather than freely move around as in gas or liquids. As you lower temperature, the amplitude of vibration and consequently the vibration speed decrease, and the molecules eventually cease to move at absolute zero. However, modern quantum physics tells that the classical picture given here has a limitation. If you go down to very low temperature, the physical phenomena cannot be fully described by the classical laws and warrant a novel description, quantum physics. According to quantum physics, even at absolute zero, constituent particles still vibrate, which is called *zero-point motion*. From the classical point of view, it is proper to say that all particles cease to move or vibrate at absolute zero.

Imagine that you are holding a large but light board in front of you. From the other side of the board, many small balls are thrown with a certain speed to the board and are bounced. You will feel that the board tends to move towards you. You need to apply a force to keep the balance. Furthermore, if the speed of the balls is higher, the stronger force is need to balance. Now apply this to gas contained in a cylinder as shown in the figure. All the walls are rigid immobile but the top can slide freely along the cylinder without any friction. As you pump in gas at T, the top will move up. The wall start to move because there is a force on the wall upward. That force is provided by the colliding gas molecules. Stop pumping in gas. Then the wall stays at a certain position. Now, we can specify a few physical quantities since things are not changing: temperature T, volume V, and finally

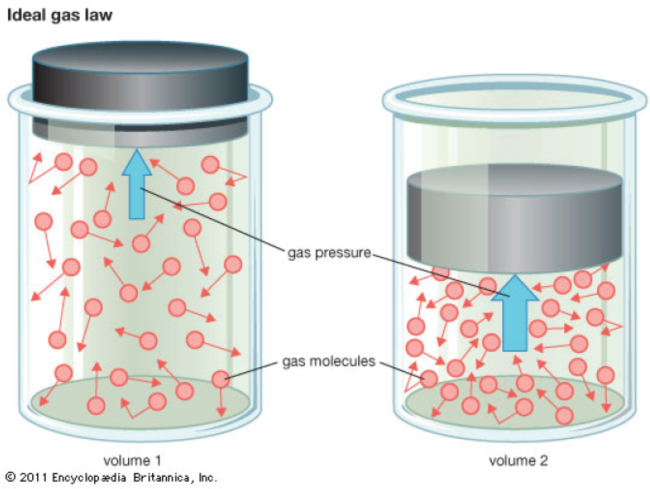

FIG. 1:

pressure P. Gas molecules inside the container constantly collide onto the wall and apply a force on it. But the wall is at rest, which means the total force on the wall should be zero. The force acting on the wall from the gas is balanced by the force by molecules in air outside of the container. We can describe this phenomena using pressure rather than force because pressure (force per unit area) does not depend on the area of the wall. Physicists discovered that there is a universal relation between T, V, and P. *Universal* is added to emphasize that the same relation holds for different types of gases. This is called *Ideal Gas Law*: $PV = Nk_BT$. Here N is the number of gas molecules in the container.

• In a fixed volume container, the pressure of gas is proportional to temperature. If you increase temperature from 300 K to 600 K, the pressure of gas doubles. $P \propto T$

• If you keep the pressure constant and allow the volume change, then volume is proportional to temperature. $V \propto T$

• Keep the temperature constant. If you reduce the volume of gas by pushing down the top wall in the figure, the pressure of the gas increases inversely. $P \propto 1/V$

Exercise One mole of gas is contained in a 1 liter volume. One can calculate the pressure of the gas at 300 K. First of all, convert all the quantities in proper units. 1 mole is 6×10^{23} molecules. 1 liter is 10^{-3} m^3. Then $P = Nk_BT/V = (6 \times 10^{23})(1.4 \times 10^{-23})(300)/0.001 = 2.52 \times 10^6$ Pa. It is 25.2 times of atmospheric pressure since 10^5 Pa is 1 atm (atmosphere).

The apparatus shown in the figure describes the situation of constant pressure (1 atm) since the moving wall (cap) is in balance at atmospheric pressure. If you change the temperature of gas, the volume will change (the cap will move up or down) in accordance with ideal gas law. Let us do a simple experiment. Vary the temperature of the apparatus and record several sets of T and V. Plot T (x-axis) vs V (y-axis). Then $V = Nk_BT/P$ suggests you should be able to fit the data with a straight line, and from the slope you can calculate the number of molecules in the container! We will do a slightly different experiment today.